

GReCO seminar, IAP
October 2022, 10th, Paris

Based on:

[Fumagalli, Garcia-Saenz, LP, Renaux-Petel, Ronayne 2019]

Phys. Rev. Lett. 123, 201302

[Garcia-Saenz, LP, Renaux-Petel 2020]

J. High Energ. Phys. 2020, 73 (2020)

[LP 2020]

J. Cosm. & Astro. Phys. 04(2021)048

[LP, Aoki, Renaux-Petel, Yamaguchi 2021]

ArXiv:2112.05710

[Dimastrogiovanni, Fasiello, LP 2022]

JCAP 09 (2022) 031

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

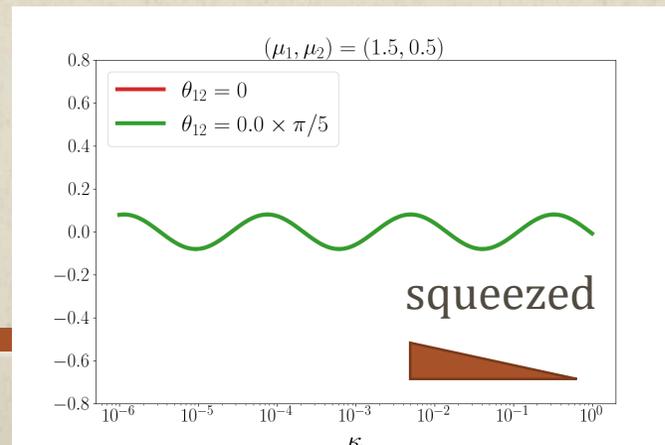
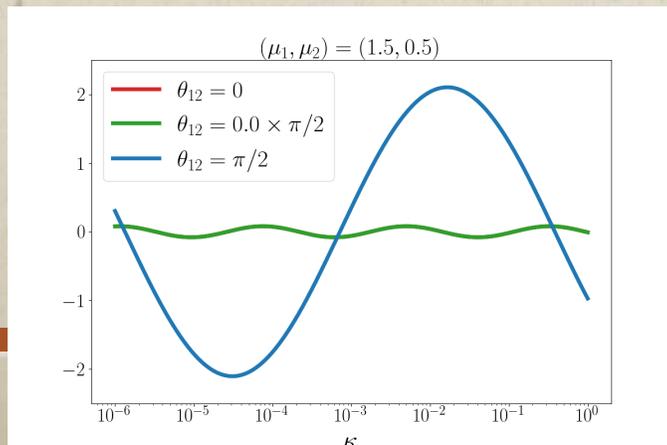
ArXiv: 2207.14267



Lucas Pinol

Instituto de Física Teórica (IFT) UAM-CSIC
Madrid

THE NON-LINEAR UNIVERSE AS A PARTICLE DETECTOR



Cosmic spectroscopy:

In the squeezed limit of the primordial scalar bispectrum, modulated oscillations depend on the masses and mixing angles of the inflationary theory

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Lucas Pinol

Institut d'Astrophysique de Paris (IAP)

GReCO seminar, IAP
June 2021, virtual

Lucas Pinol
With S. Renaux-Petel, Y. Tada
Institut d'Astrophysique de Paris (IAP)

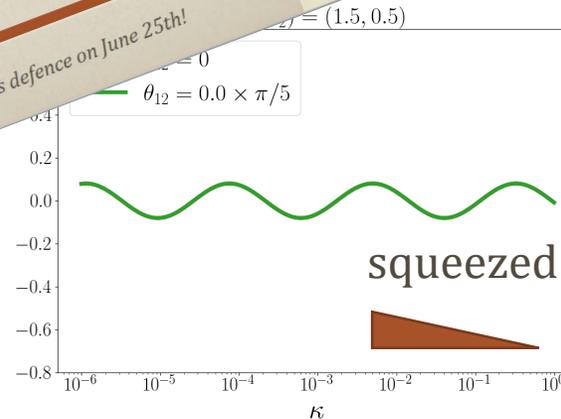
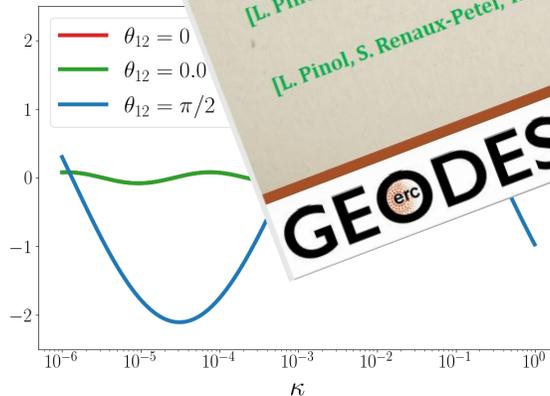
GEOMETRICAL ASPECTS OF STOCHASTIC INFLATION
A PATH (INTEGRAL) TO THE DISCRETISATION AMBIGUITY AND ITS RESOLUTION

[L. Pinol, S. Renaux-Petel, Y. Tada 2018] *Classical and Quantum Gravity* 36 no.7, (2019) 07LT01
CQG Highlights of 2019-2020

[L. Pinol, S. Renaux-Petel, Y. Tada 2020] *Journal of Cosmology and Astroparticle Physics*, JCAP04(2021)048

GEODESI

Thesis defence on June 25th!



Cosmic spectroscopy:

In the squeezed limit of the primordial scalar bispectrum, modulated oscillations depend on the masses and mixing angles of the inflationary theory

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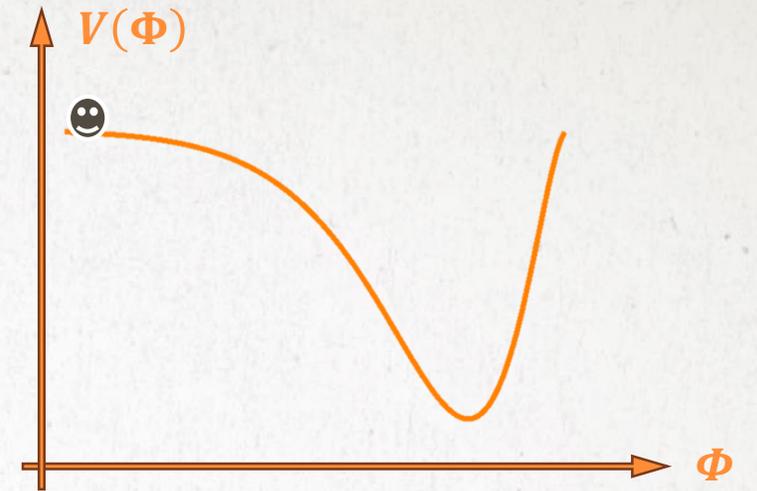
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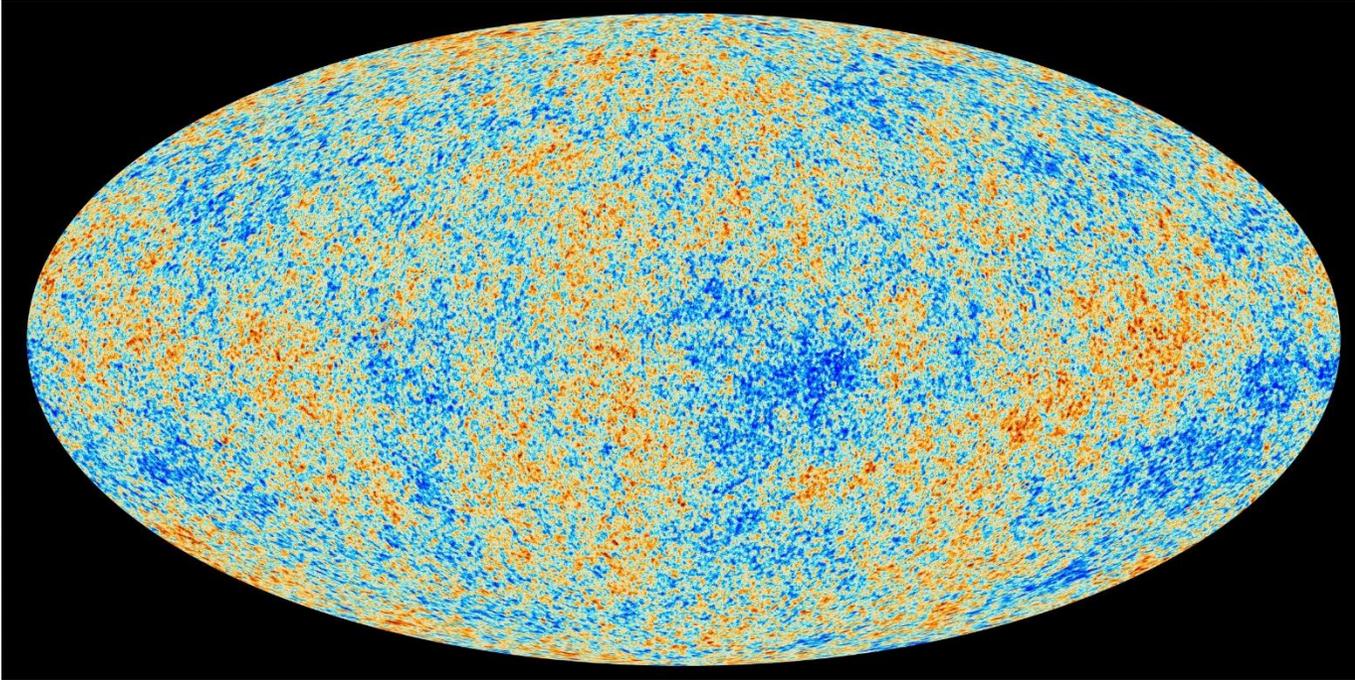
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I. INFLATION

Success story, missing evidence
and future prospects



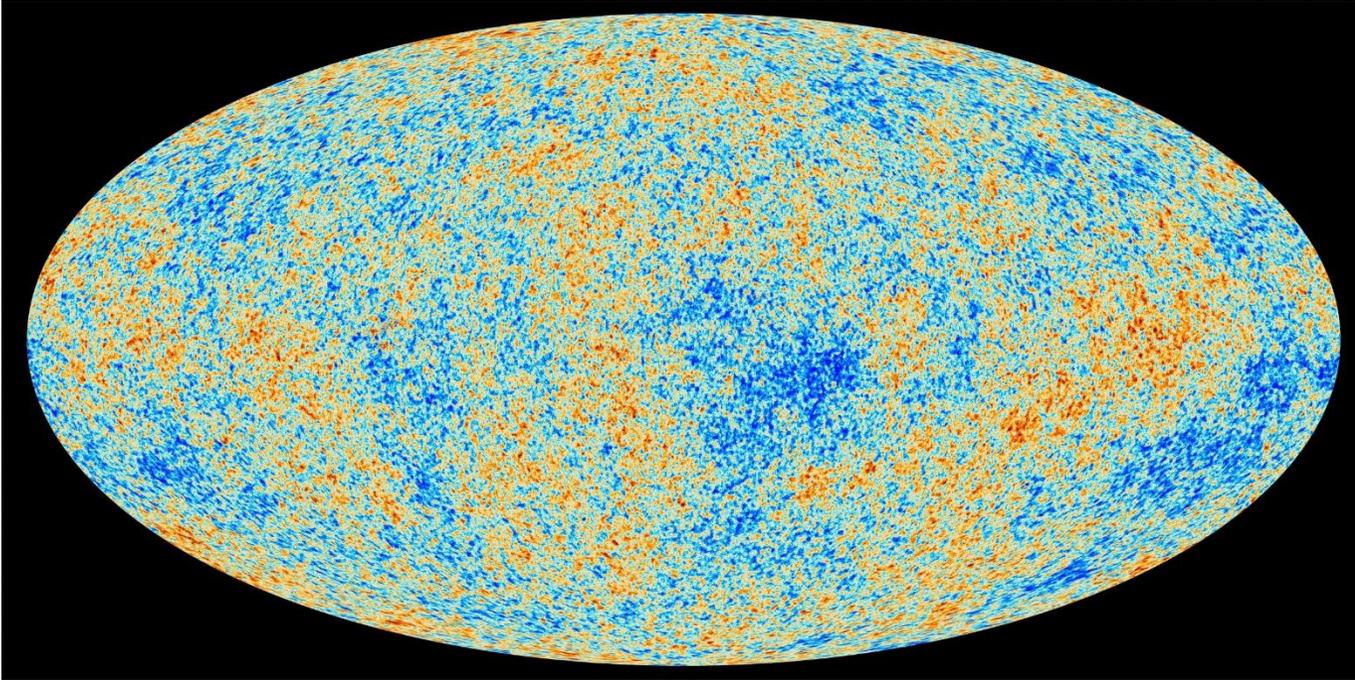
CMB OBSERVATION MOTIVATES INFLATION



$$T \sim 2.73K ; \frac{\delta T}{T} \sim 10^{-5} ; |\Omega_K| \ll 1$$

- How is the universe so homogeneous?
Horizon problem
- Why is the universe so spatially flat?
Flatness problem

CMB OBSERVATION MOTIVATES INFLATION



$$T \sim 2.73K ; \frac{\delta T}{T} \sim 10^{-5} ; |\Omega_K| \ll 1$$

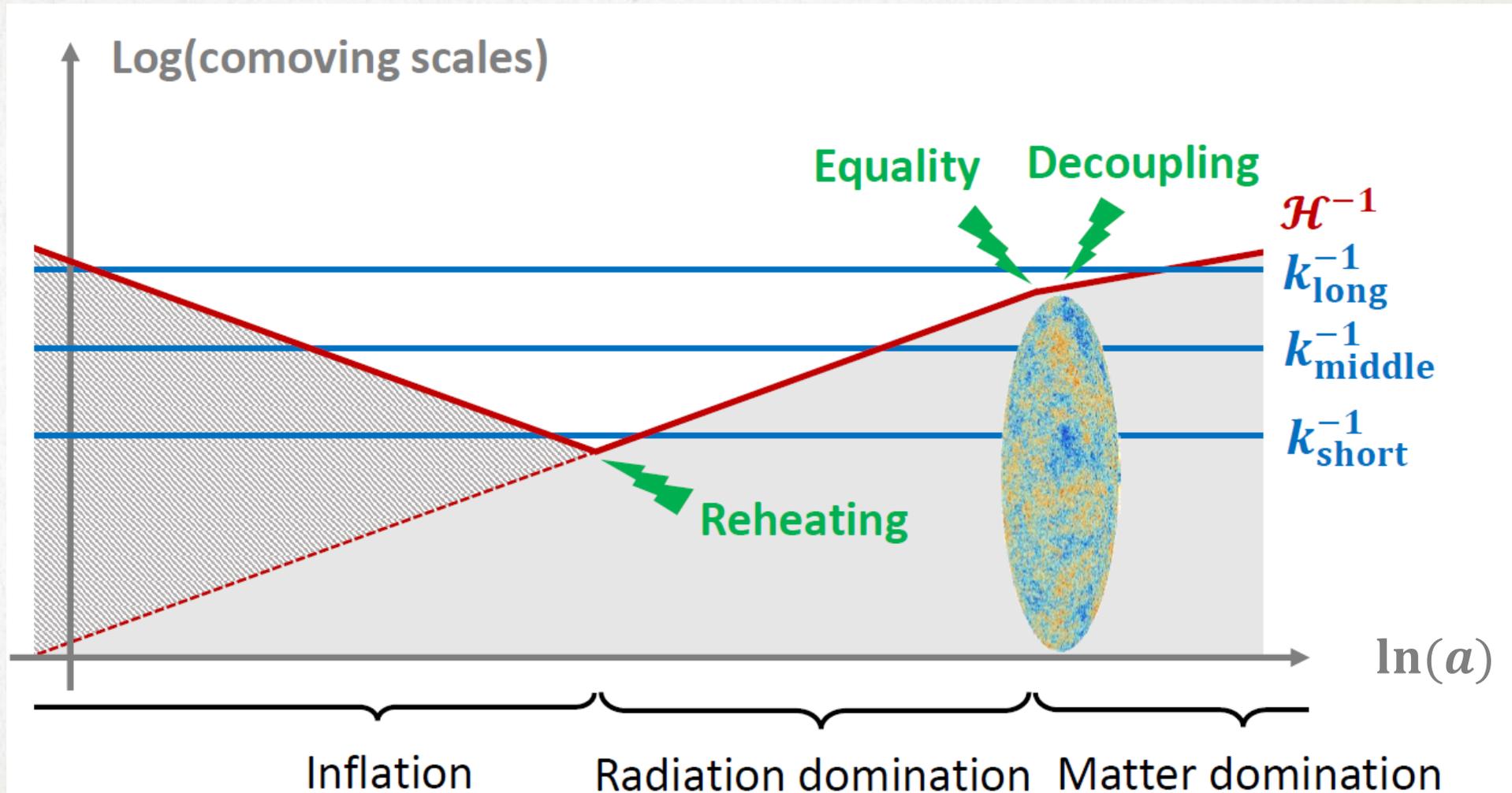
- How is the universe so homogeneous?
Horizon problem
- Why is the universe so spatially flat?
Flatness problem

**Inflation, an era of accelerated expansion of the Universe,
solves both the horizon and flatness problems**

$$N_{\text{inf}} = \ln \left(\frac{a_{\text{end}}}{a_{\text{ini}}} \right) \gtrsim 55$$

FLUCTUATIONS OF MICRO-PHYSICAL QUANTUM ORIGIN

$\mathcal{H}^{-1} = (aH)^{-1}$
 Comoving Hubble
 radius
 $\tau = \int d \ln a \mathcal{H}^{-1}$
 proper time
 =
 particle horizon

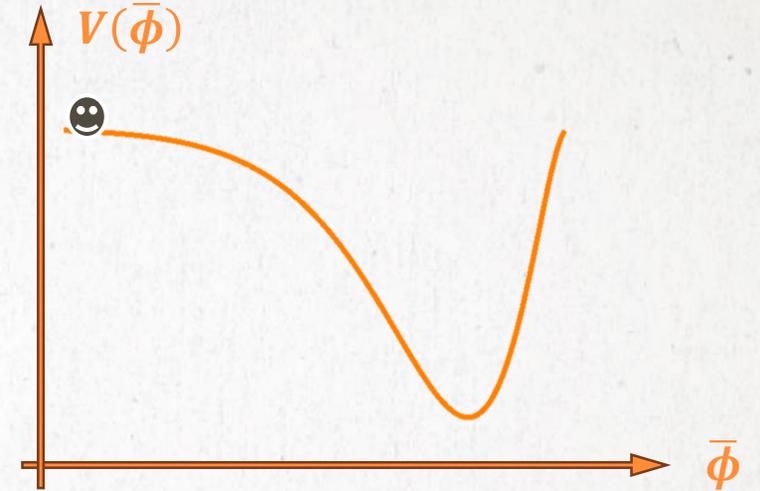


MECHANICS OF INFLATION: CURRENT PARADIGM

A single **scalar field** in slow roll does the job for both:

- The classical **background**...
- The quantum **fluctuations**...

$$\phi(\vec{x}, t) = \bar{\phi}(t) + Q(\vec{x}, t)$$



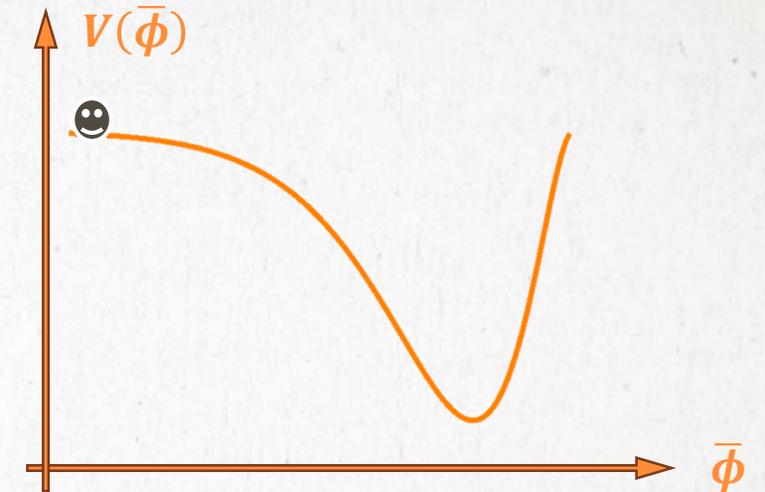
MECHANICS OF INFLATION: CURRENT PARADIGM

A single scalar field in slow roll does the job for both:

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... provided the scalar potential is flat and inflation lasts long enough

- The quantum fluctuations...



$$\phi(\vec{x}, t) = \bar{\phi}(t) + Q(\vec{x}, t) \quad \text{with } Q(\vec{x}, t) \ll \bar{\phi}(t)$$

Homogeneous background, slow roll: $\dot{\bar{\phi}} \simeq -\frac{V_{,\phi}(\bar{\phi})}{3H}$; $H^2 \simeq \frac{V(\bar{\phi})}{3M_{\text{Pl}}^2}$

CLASSICAL

MECHANICS OF INFLATION: CURRENT PARADIGM

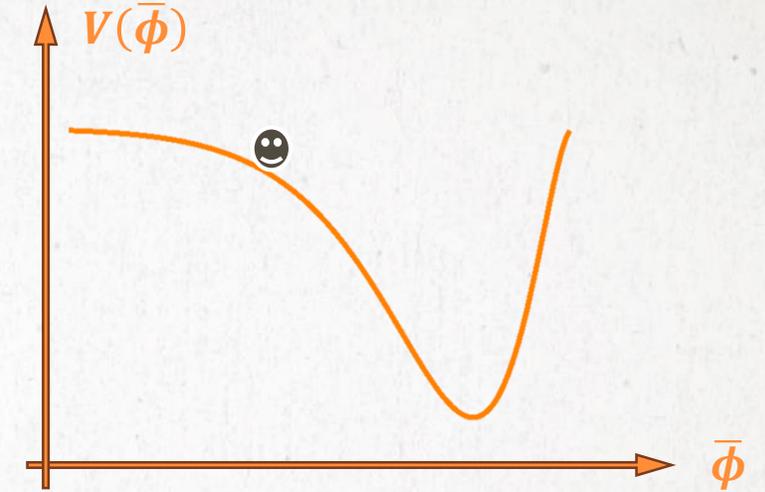
A single scalar field in slow roll does the job for both:

- The classical background...

... provided the scalar potential is flat and inflation lasts long enough

- The quantum **fluctuations**...

... if they emerge from the Bunch-Davies (BD) vacuum



$$\phi(\vec{x}, t) = \bar{\phi}(t) + Q(\vec{x}, t) \quad \leftarrow \text{Bunch-Davies: } Q_k(\tau) \simeq \frac{e^{-ik\tau}}{a\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right) \rightarrow \zeta_k(\tau) \simeq \frac{H}{\dot{\phi}} Q_k(\tau)$$

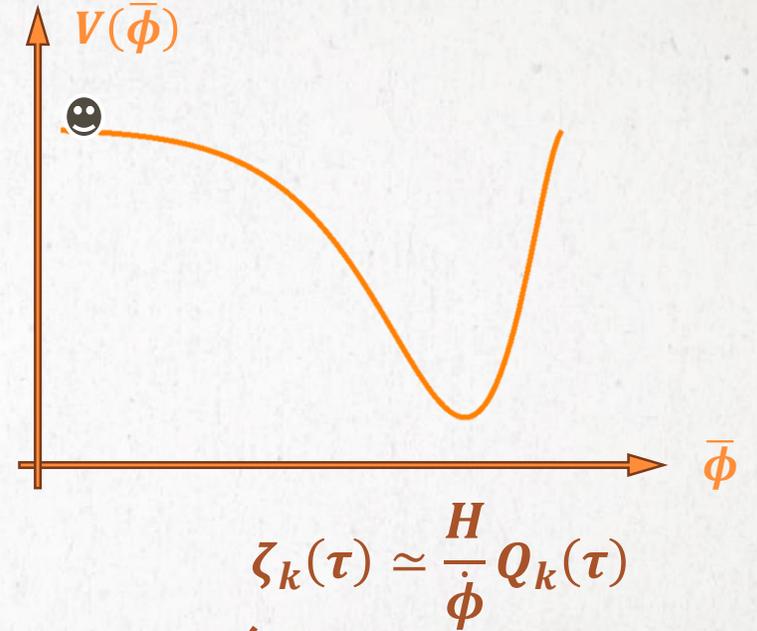
QUANTUM

Homogeneous background, slow roll: $\dot{\phi} \simeq -\frac{V_{,\phi}(\bar{\phi})}{3H}$; $H^2 \simeq \frac{V(\bar{\phi})}{3M_{\text{Pl}}^2}$ CLASSICAL

MECHANICS OF INFLATION: CURRENT PARADIGM

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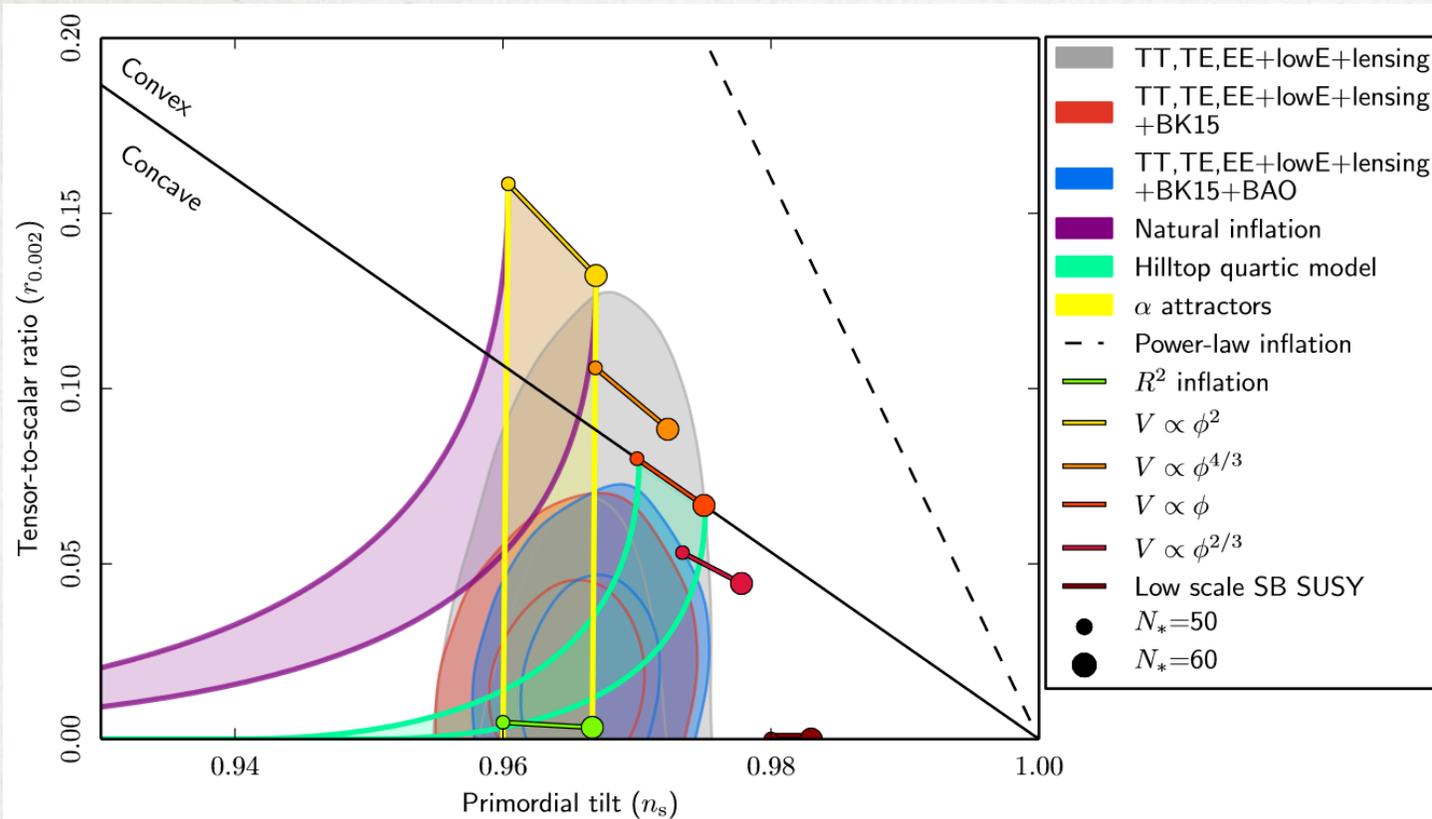
Homogeneous background, slow roll: $\dot{\phi} \simeq -\frac{V_{,\phi}(\bar{\phi})}{3H}$

Almost scale-invariant power spectrum: $n_s \simeq 1$

Also, $\mathcal{P}_\gamma(k) = r A_s \left(\frac{k}{k_*}\right)^{n_t}$, with $r \ll 1$

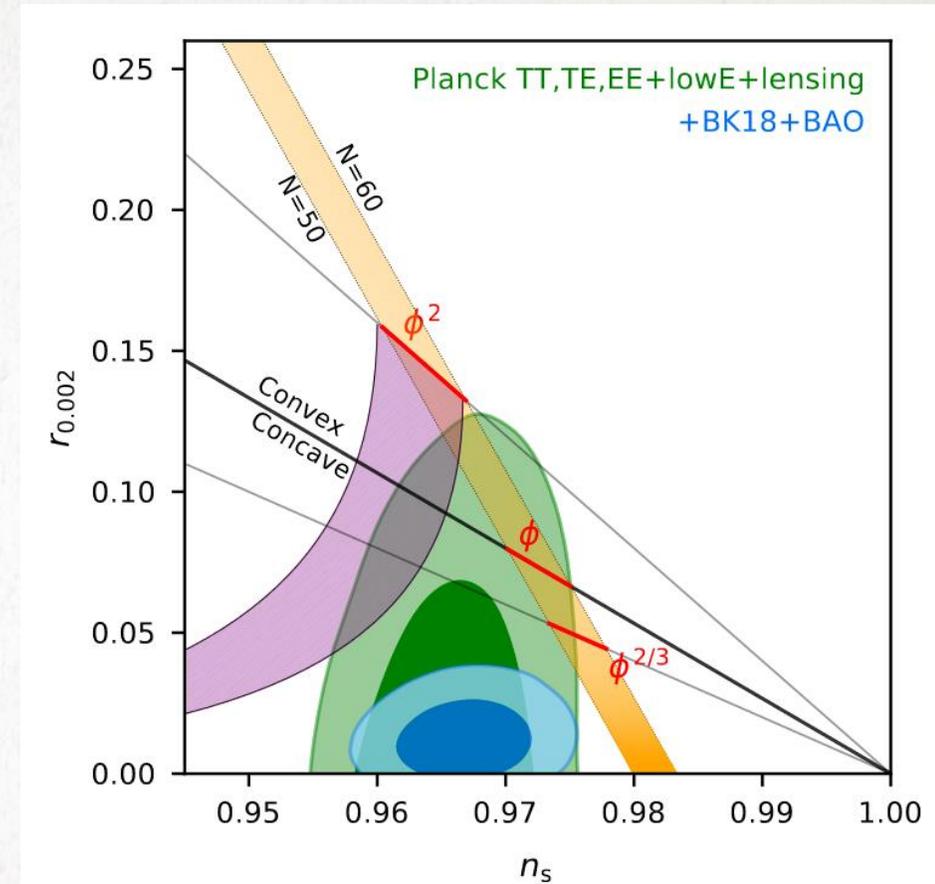
$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$

SINGLE-FIELD INFLATION: CONSTRAINTS



[Planck 2018]

$$n_s = 0.9649 \pm 0.0042$$



[BICEP-Keck 2022]

$$r < 0.032$$

MISSING EVIDENCE AND FUTURE PROSPECTS

- Primordial tensor modes of quantum-mechanical origin are expected *at some level*

Key prediction of inflation as a theory of gravity with quantized linear fluctuations + vacuum initial conditions

*Many upcoming experiments to be much more sensitive on B modes in the CMB:
Simons Observatory, LiteBIRD, CMB-S4 → $r < \mathcal{O}(10^{-3})$*

- Primordial non-Gaussianities have the same status and have not been observed yet neither

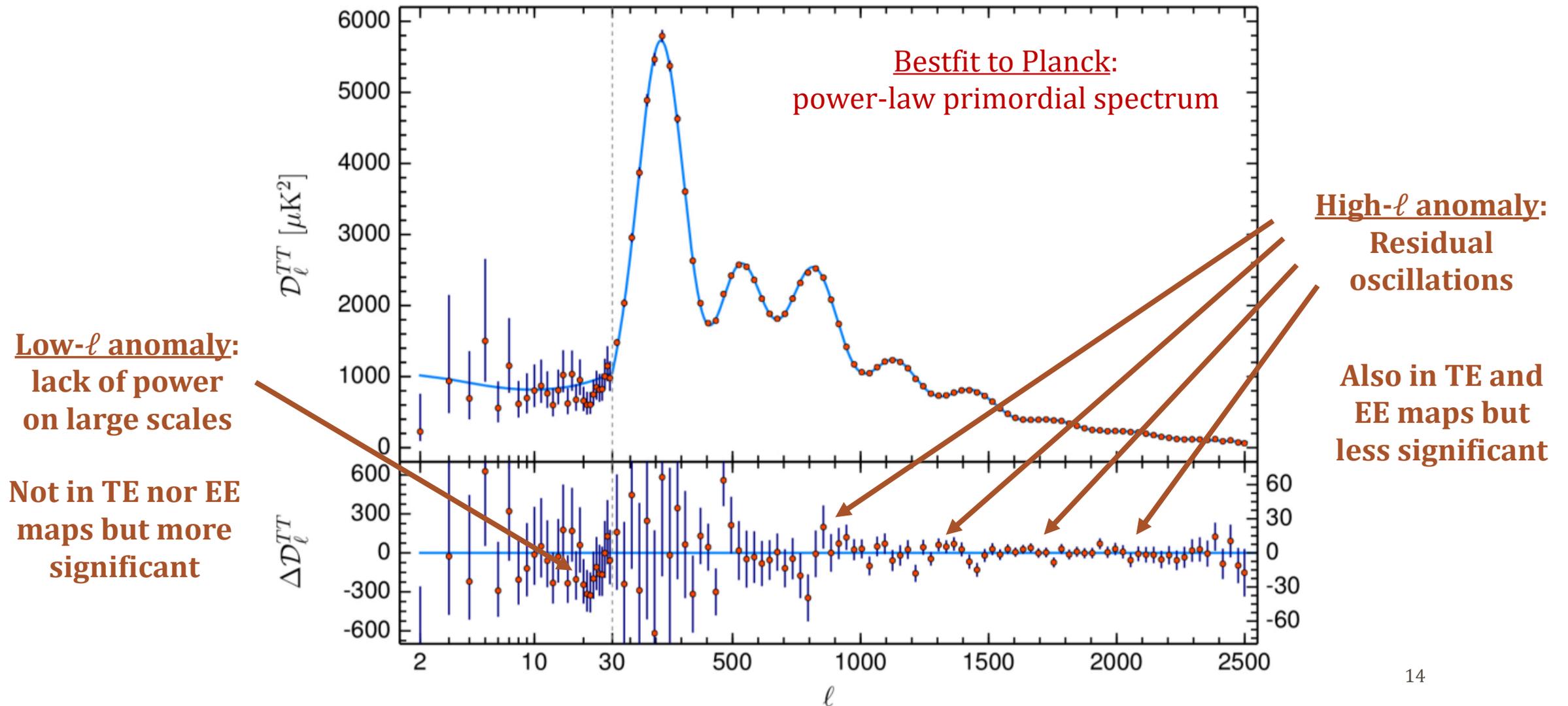
Gravitational interactions always present + non-trivial multifield interactions give PNG *at some level*

*More challenging: CMB limited by cosmic variance. There is hope with LSS experiments:
DESI, Euclid, SPHEREx → $f_{\text{NL}} < \mathcal{O}(1)$; Brightness temperature 21-cm maps → $f_{\text{NL}} < \mathcal{O}(0.1)$*

- Theoretical status: few theoretical motivation for single scalar field with a flat potential
Inflation with a shift-symmetry, with non-minimal coupling, multifield inflation, curved field space, etc.

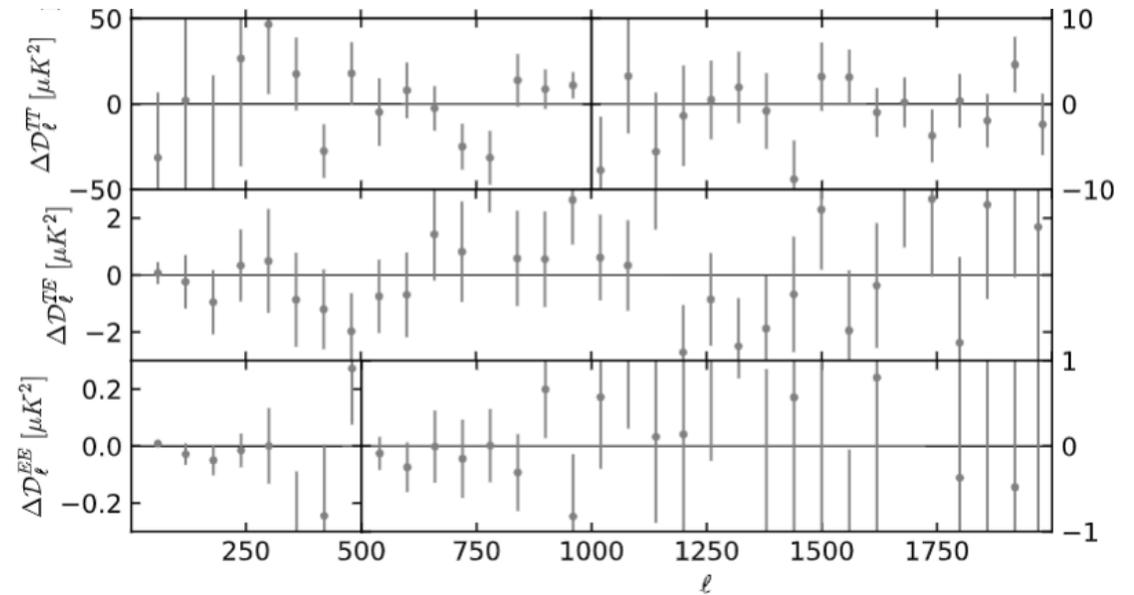
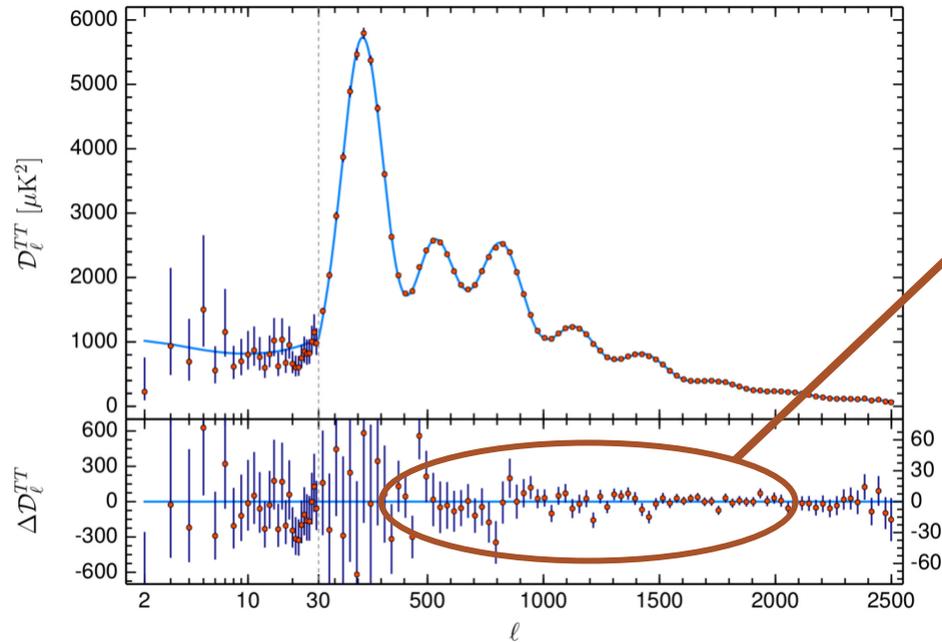
ANOMALIES / RESIDUALS / BEYOND POWER LAW

CMB temperature and polarisation maps (TT, TE, EE) contain residuals not fitted by the standard picture



ANOMALIES / RESIDUALS / BEYOND POWER LAW

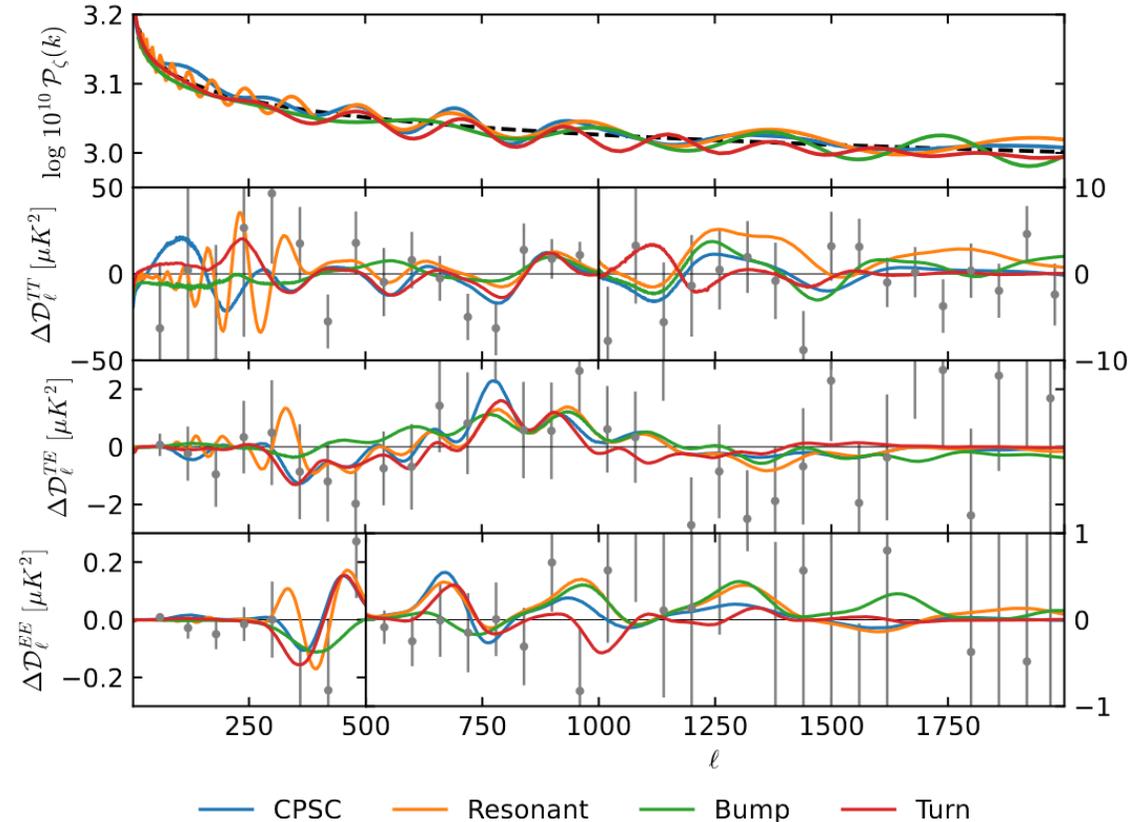
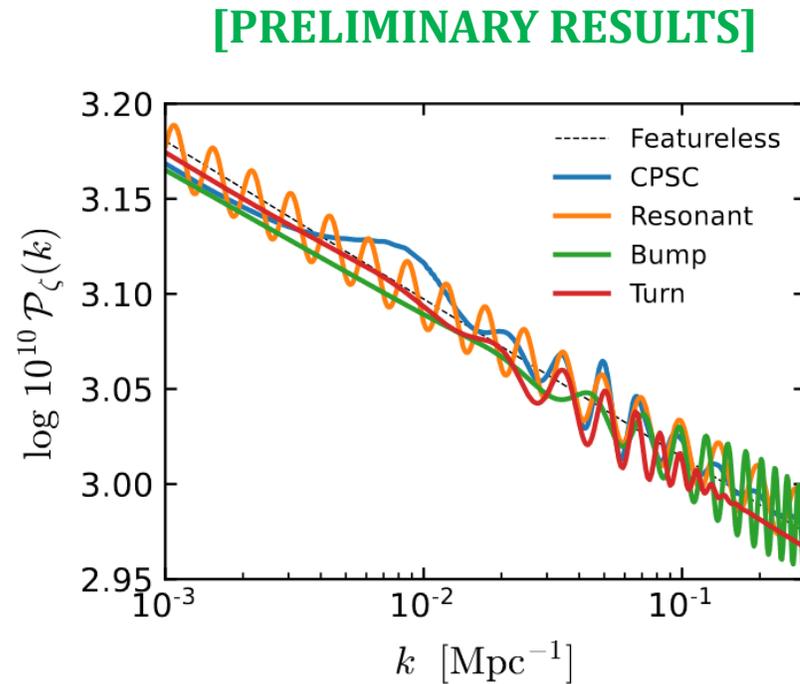
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[Credits Matteo Braglia]

ANOMALIES / RESIDUALS / BEYOND POWER LAW

CMB temperature and polarisation maps (TT, TE, EE) contain residuals not fitted by the standard picture



[Braglia, Chen, Hazra, Pinol in prep.]

	CPSC	Resonant	Bump	Turn
$\ln B$	-1.2 ± 0.36	-2.24 ± 0.38	-1.44 ± 0.36	-2.31 ± 0.36
$\Delta \chi^2$	13.4	10.9	8.9	8.6

→ Statistically slightly disfavored

→ Better fit

ANOMALIES / RESIDUALS / BEYOND POWER LAW

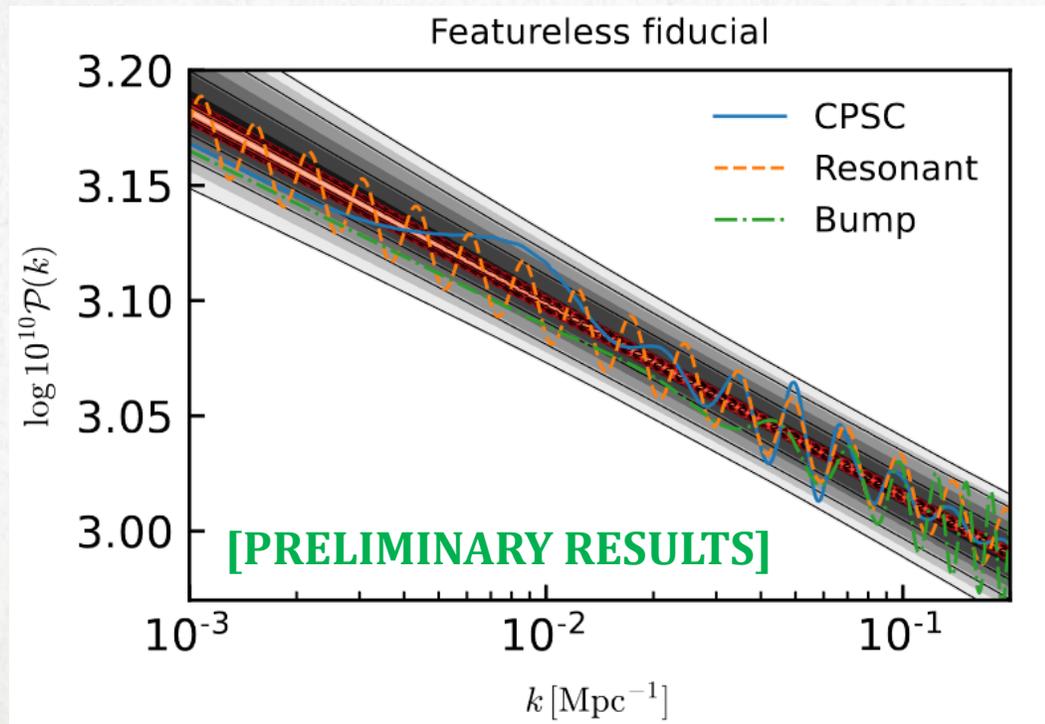
Future “B-modes” experiments (thanks to much better E-modes) like LiteBIRD, SO, CMB-S4 will tell!

- If the universe is featureless: current bestfits (turn, bump, CPSC, resonant, ...) will all be ruled out
[PRELIMINARY RESULTS]

ANOMALIES / RESIDUALS / BEYOND POWER LAW

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Grey: Planck error bars (data)

Red: PL+LB+S4 error bars (forecast)

ANOMALIES / RESIDUALS / BEYOND POWER LAW

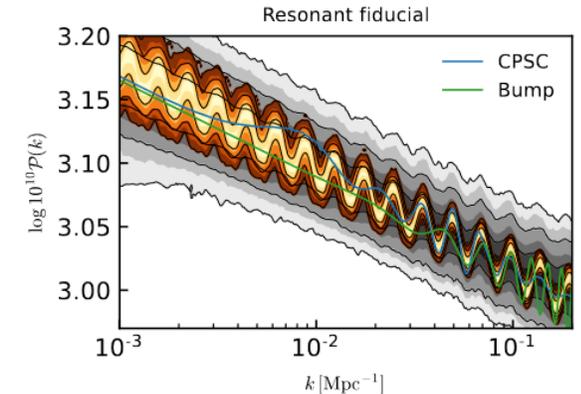
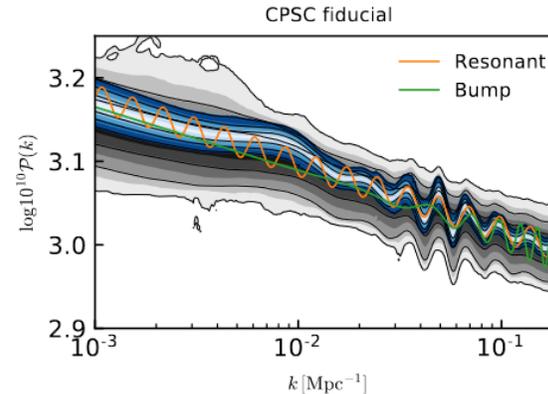
Future “B-modes” experiments (thanks to much better E-modes) like LiteBIRD, SO, CMB-S4 will tell!

- If the universe is featureless: current bestfits (turn, bump, CPSC, resonant, ...) will all be ruled out
- If one of the current feature bestfit indeed represents our universe:
 - ❖ The featureless universe will always be ruled out
 - [PRELIMINARY RESULTS]**
 - ❖ We will tell apart different feature models (often)

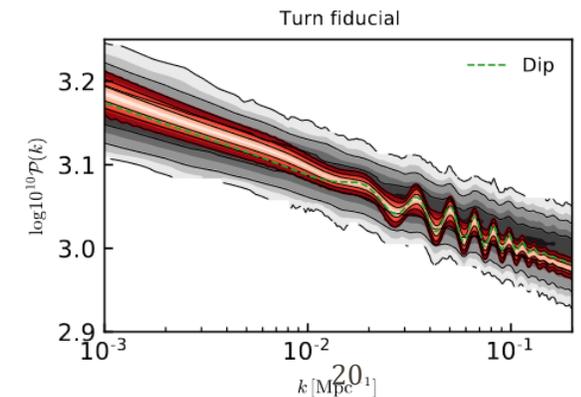
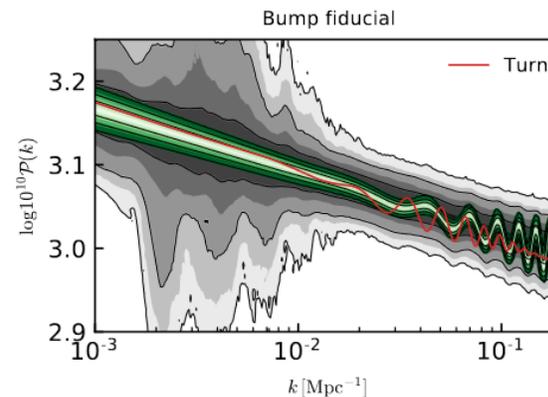
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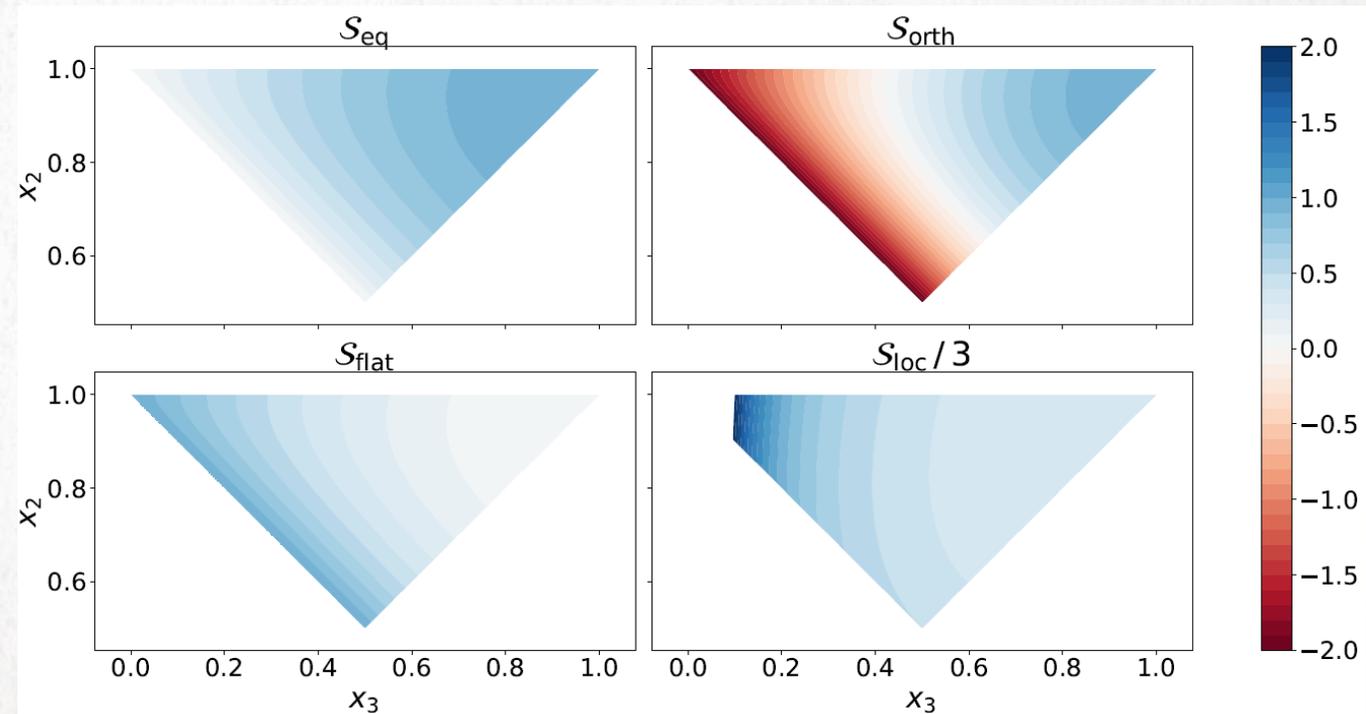


[PRELIMINARY RESULTS]



II. PRIMORDIAL NON-GAUSSIANITIES AS A PROBE OF THE SCALAR CONTENT

Single-field inflation
Multifield inflation
The cosmic spectroscopy



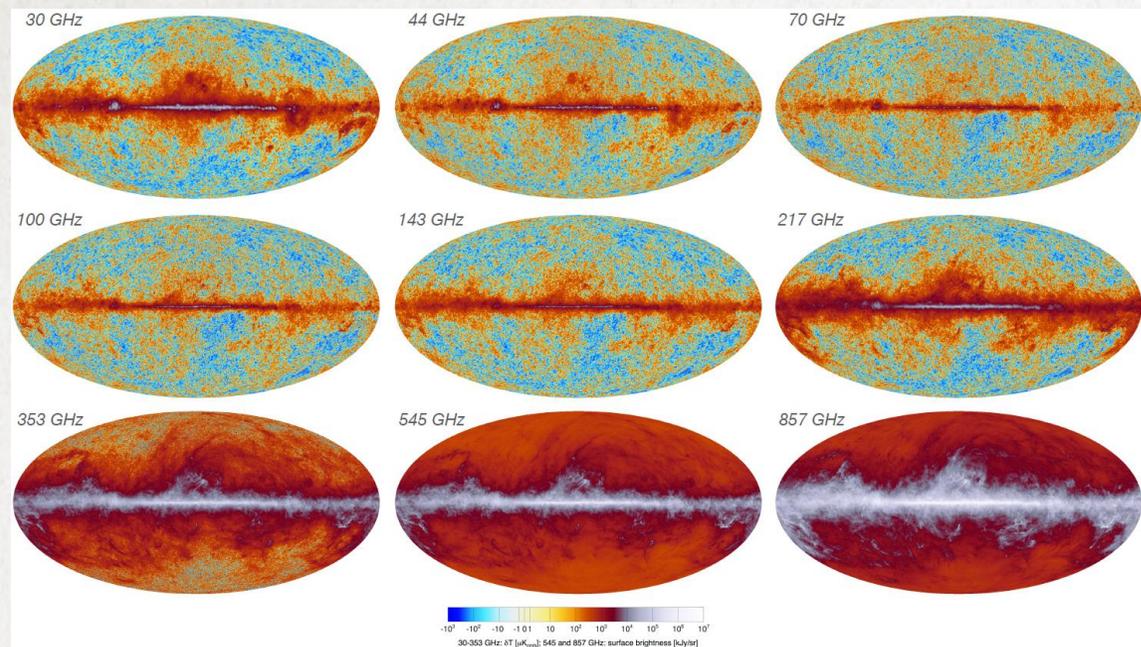
Single-field inflation (and definitions)

Non-linearities in the sky

Sources of non-Gaussianity:

- Foreground
- Late-time evolution: lensing, etc.
- Early-time evolution: gravity, interactions, etc.
- **Initial conditions:**

Primordial non-Gaussianities from inflation



Planck CMB intensity maps

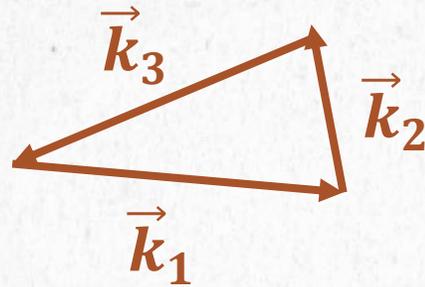
$$T_{\text{ini}}(\theta, \varphi) = T_{\text{ini}}^G(\theta, \varphi) + f_{\text{NL}}^{\text{local}} \times [T_{\text{ini}}^G(\theta, \varphi)]^2$$

\nearrow Gaussian \nearrow Non-Gaussian if $f_{\text{NL}}^{\text{local}} \neq 0$

PRIMORDIAL BISPECTRUM

ζ the primordial curvature perturbation

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^7 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \frac{A_s^2}{(k_1 k_2 k_3)^2} \times S(k_1, k_2, k_3)$$



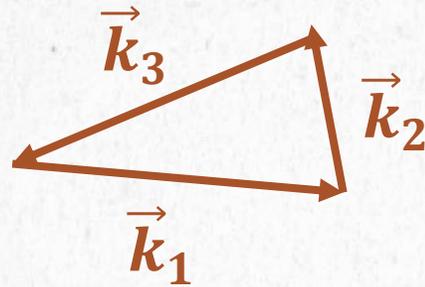
Power spectrum = 2.10×10^{-9}

Shape function

PRIMORDIAL BISPECTRUM

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Power spectrum = 2.10×10^{-9}

Shape function

< 0.0035 (from $r < 0.056$)

[Maldacena 2003]

Ex: Single-field inflation
(attractor)

$$S = \frac{5}{12} (1 - n_s) S_{\text{loc}} + \frac{\epsilon}{8} S_{\text{eq}} + \dots = \text{VERY SMALL}$$

0.0015

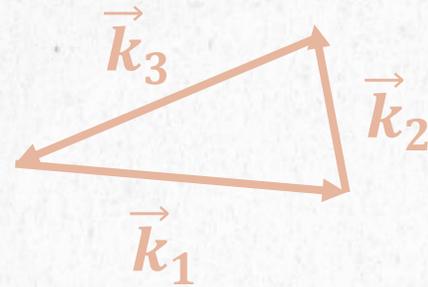
PRIMORDIAL BISPECTRUM

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Power spectrum = 2.10×10^{-9}

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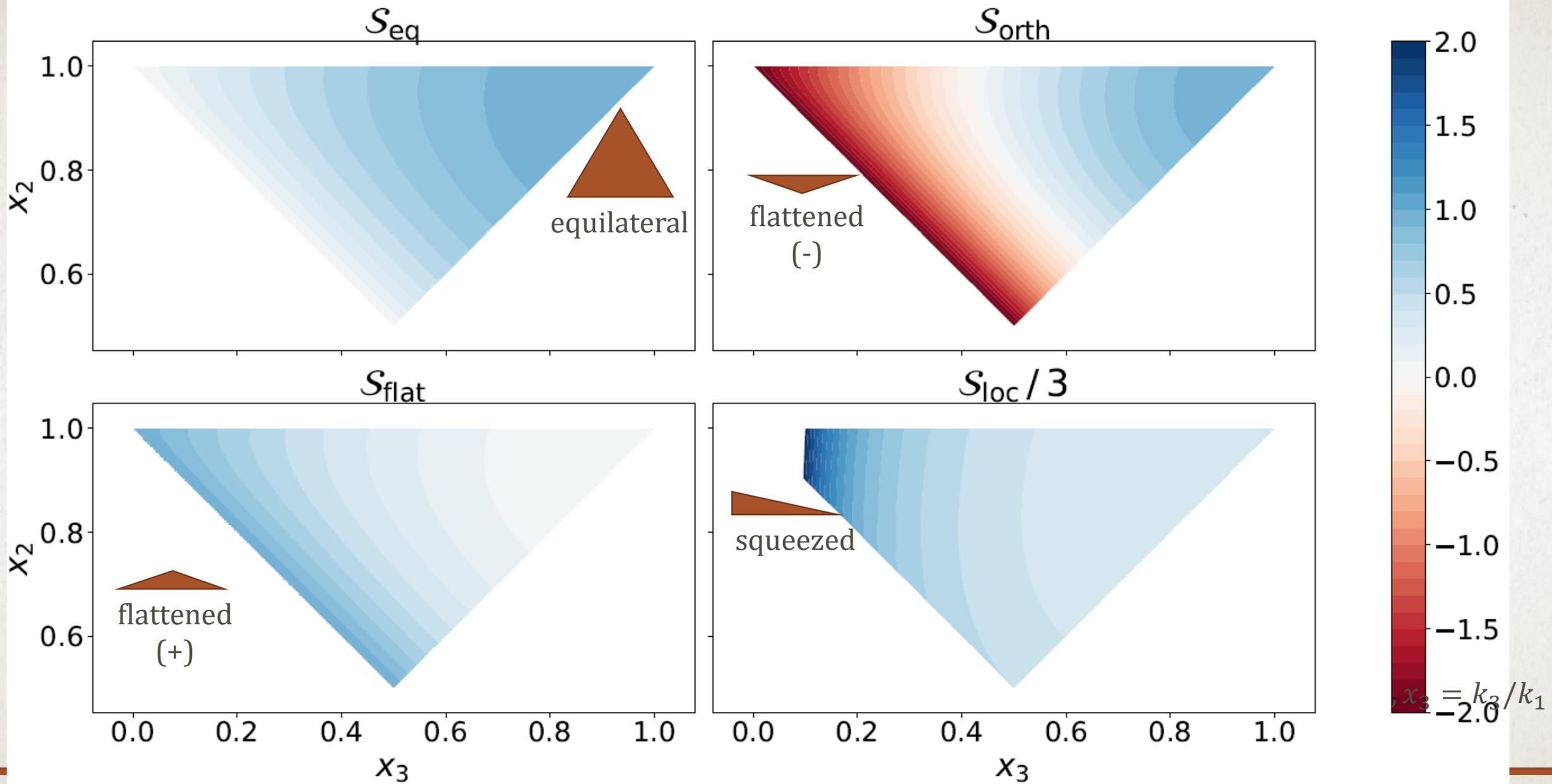
Shape templates

Ex: Single-field inflation
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$f_{\text{NL}}^{\text{loc}}$

$f_{\text{NL}}^{\text{eq}}$



OBSERVATIONAL CONSTRAINTS

$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1$$

$$f_{\text{NL}}^{\text{eq}} = -26 \pm 47$$

$$f_{\text{NL}}^{\text{ortho}} = -38 \pm 24$$

[Planck 2018]

Ex: Single-field inflation
(attractor)

Shape templates

$$S = \frac{5}{12} (1 - n_s) S_{\text{loc}} + \frac{\epsilon}{8} S_{\text{eq}} + \dots = \text{VERY SMALL}$$

$f_{\text{NL}}^{\text{loc}}$

$f_{\text{NL}}^{\text{eq}}$

SINGLE-FIELD EFFECTIVE THEORY OF INFLATIONARY FLUCTUATIONS

EFT of broken time-diffeomorphisms “beyond models”:

$$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, g^{00}, R_{\mu\nu\sigma\rho}, K_{ij}, \nabla_\mu; t)$$

[Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007]

↓
Tadpole cancellation
Goldstone boson in the unitary gauge
Decoupling limit, neglecting higher-order derivatives

$$S_{2,3}^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 \epsilon M_{\text{Pl}}^2 \left\{ \left(\frac{\zeta'^2}{c_s^2} - (\partial_i \zeta)^2 \right) + \frac{1}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right) \right\}$$

- ❖ c_s is the speed of sound
- ❖ A parameterizes the relative size of cubic derivative interactions

Non-linearly realized symmetry:
Same operator produces both quadratic and cubic interactions

Like a Wilson coefficient: naturally of order unity

SINGLE-FIELD EFFECTIVE THEORY OF INFLATIONARY FLUCTUATIONS

EFT of broken time-diffeomorphisms “beyond models”:

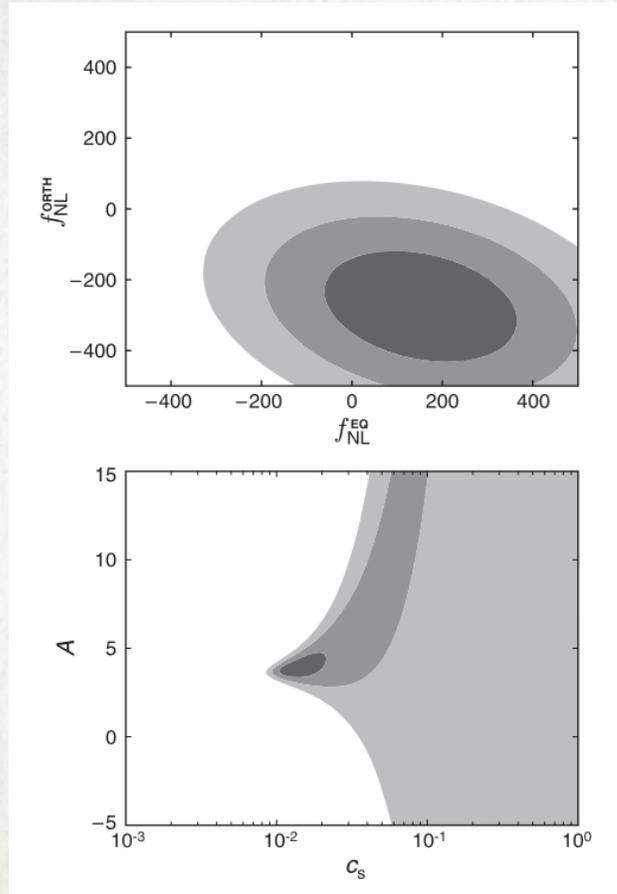
$$\mathcal{P}_\zeta = \frac{H_\star^2}{8\pi^2 \epsilon_\star c_{S\star} M_{\text{Pl}}^2} \begin{cases} n_s - 1 = -2\epsilon_\star - \eta_\star - s_\star \text{ with } s = \dot{c}_s / (Hc_s) \\ r = 16\epsilon_\star c_{S\star} \end{cases}$$

$$S = \left(\frac{1}{c_s^2} - 1 \right) \left[S_{\zeta'(\partial_i \zeta)^2} + \frac{A}{c_s^2} S_{\zeta'^3} \right] \begin{cases} f_{\text{NL}}^{\text{eq}} \simeq \frac{1}{18} \left(\frac{1}{c_s^2} - 1 \right) \left(A - \frac{17}{4} \right) \\ f_{\text{NL}}^{\text{orth}} \simeq -\frac{3}{64} \left(\frac{1}{c_s^2} - 1 \right) (1 - A) \end{cases}$$

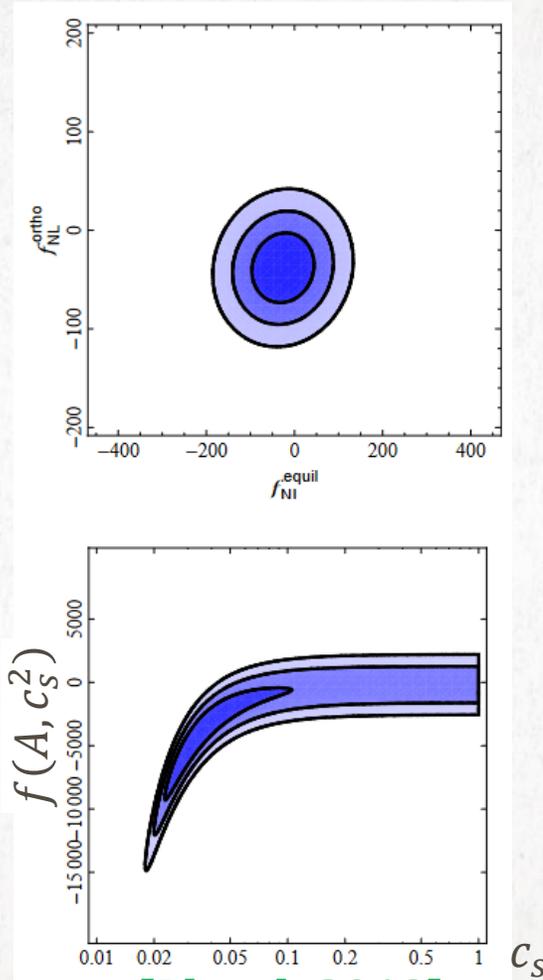
*both shapes are ~equilateral
but cancel each other for
 $A \simeq 4 \rightarrow$ ortogonal shape*

Large equilateral /
ortogonal PNGs for
small speed of
sound!

SINGLE-FIELD EFFECTIVE THEORY OF INFLATIONARY FLUCTUATIONS



[WMAP 2012]



[Planck 2019]

Large equilateral /
orthogonal PNGs for
small speed of
sound!

SINGLE-FIELD EFFECTIVE THEORY OF INFLATIONARY FLUCTUATIONS

UV-completions of the low-energy EFT:

- Single-field inflation with higher-order derivatives: P(X) inflation in slow-roll
e.g. DBI inflation gives $c_s = 1/\gamma$ "Lorentz factor" and $A = -1$
- Multifield inflation
e.g. 2-field inflation with one heavy fluctuation gives c_s , but A not known before my work
 N_{field} -inflation but nor c_s nor A known before my work



$$S_{2,3}^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 \epsilon M_{\text{Pl}}^2 \left\{ \left(\frac{\zeta'^2}{c_s^2} - (\partial_i \zeta)^2 \right) + \frac{1}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right) \right\}$$

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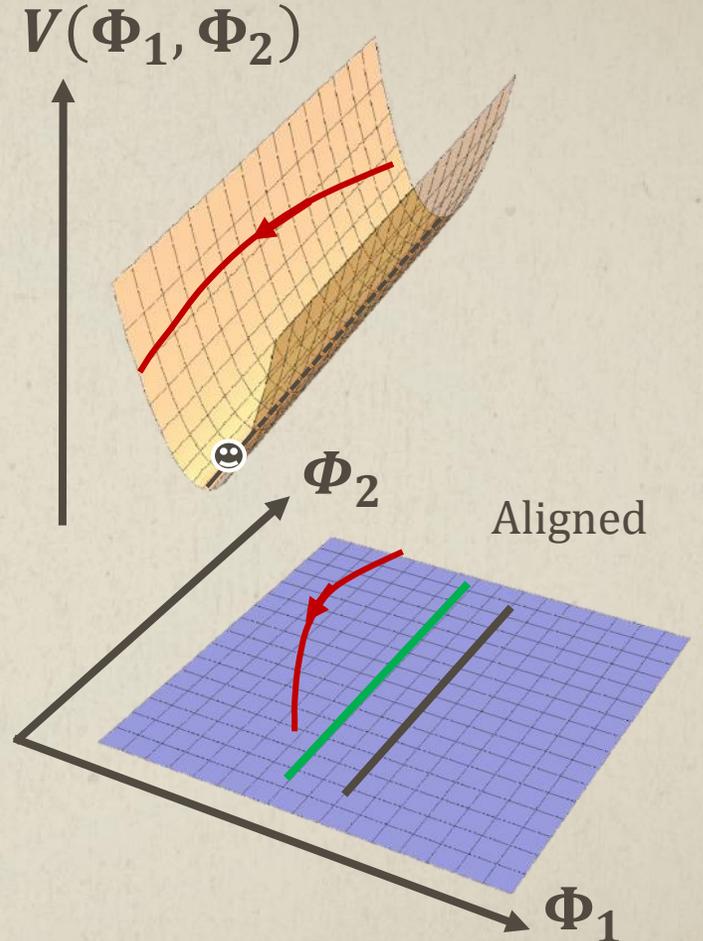
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Multifield inflation

MULTIFIELD INFLATION

$$S = \int \sqrt{-g} \left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} \delta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^c) \right)$$

- One geodesic
- Non-geodesic motion
- Minimum of the potential

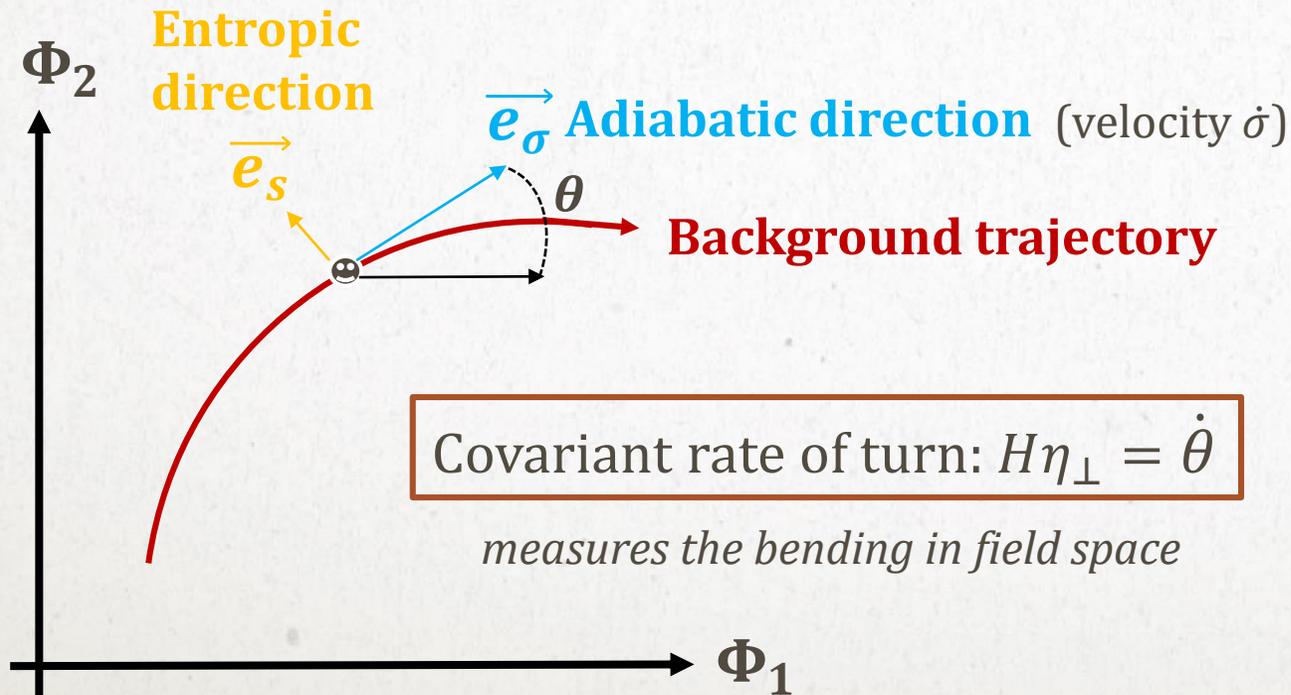


Flat field space

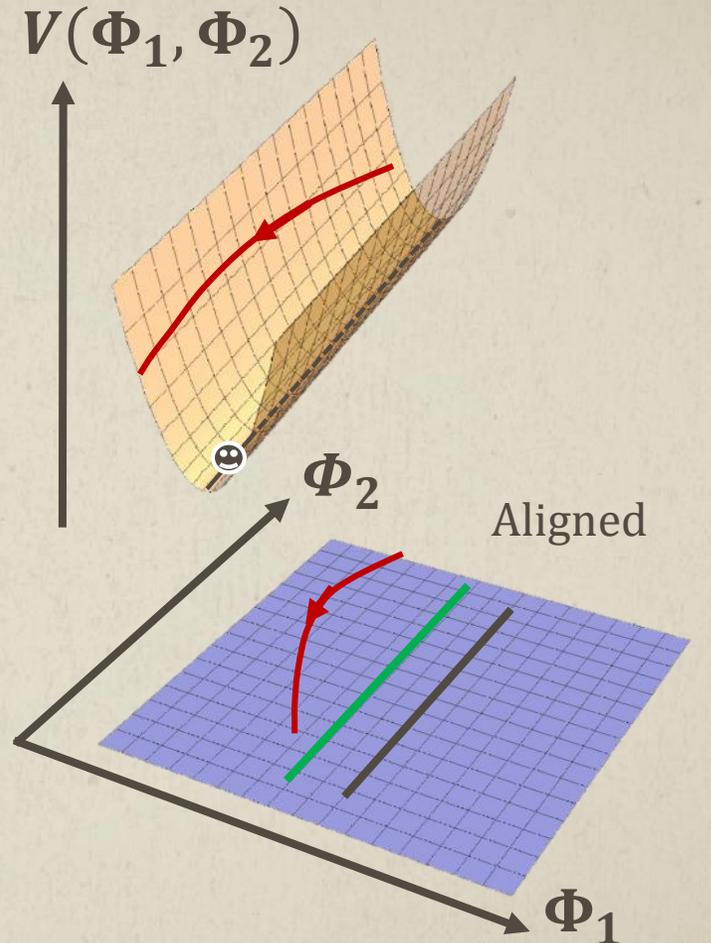
Vanishing curvature: $R_{fs} = 0$

MULTIFIELD INFLATION

$$S = \int \sqrt{-g} \left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} \delta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^c) \right)$$



- One geodesic
- Non-geodesic motion
- Minimum of the potential



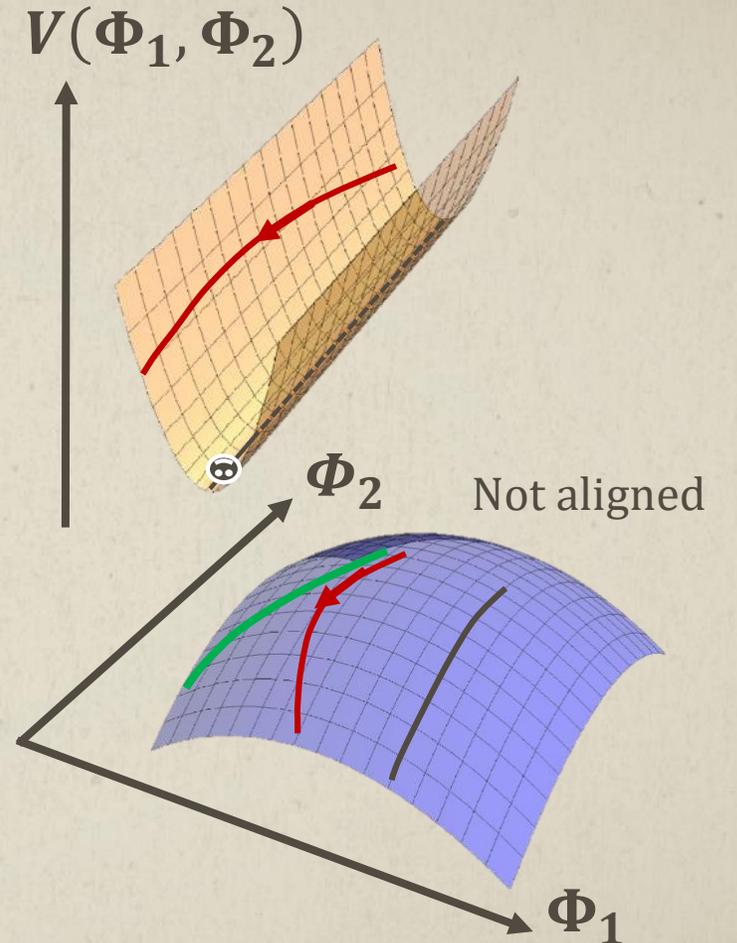
Flat field space

Vanishing curvature: $R_{fs} = 0$

MULTIFIELD INFLATION WITH CURVED FIELD SPACE

$$S = \int \sqrt{-g} \left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} G_{ab}(\phi^c) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^c) \right)$$

- One geodesic
- Non-geodesic motion
- Minimum of the potential



Curved field space

Scalar curvature: $R_{fs} \neq 0$

MULTIFIELD INFLATION WITH CURVED FIELD SPACE

$$S = \int \sqrt{-g} \left(\frac{R}{2} - \sum_{a,b} g^{\mu\nu} G_{ab}(\phi^c) \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^c) \right)$$

$$\text{Covariant rate of turn: } H\eta_\perp = D_t e_\sigma^I / e_s^I$$

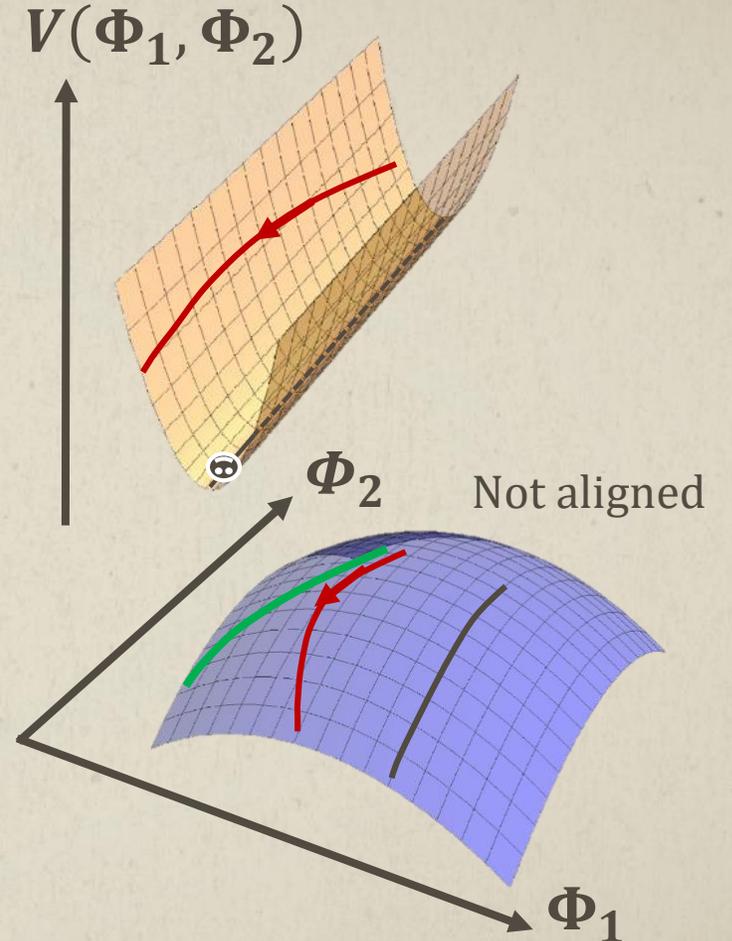
measures deviation from a geodesic in field space

Local curvature in field space

Ricci scalar R_{fs} constructed from G

Geometry	Flat	Spherical	Hyperbolic
R_{fs}	0	> 0	< 0

- One geodesic
- Non-geodesic motion
- Minimum of the potential



Curved field space

Scalar curvature: $R_{fs} \neq 0$

$$\frac{\partial^2 \chi}{a^2} = \epsilon \dot{\zeta} + \frac{\dot{\sigma}}{M_{\text{Pl}}^2} \eta_{\perp} \mathcal{F}$$

GENERALIZING MALDACENA'S CALCULATION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

$$\mathcal{L}(\zeta, \mathcal{F}) = \mathcal{L}^{(2)}(\zeta, \mathcal{F}) + \mathcal{L}_{\text{Maldacena}}^{(3)}(\zeta, \chi) + \mathcal{L}_{\text{new}}^{(3)}(\zeta, \mathcal{F}) + \mathcal{D}$$

There are two gauge-invariant scalar fluctuating degrees of freedom:

- ζ the adiabatic curvature perturbation
- \mathcal{F} the entropic perturbation

$$\frac{\partial^2 \chi}{a^2} = \epsilon \dot{\zeta} + \frac{\dot{\sigma}}{M_{\text{Pl}}^2} \eta_{\perp} \mathcal{F}$$

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$$\mathcal{L}^{(2)}(\zeta, \mathcal{F}) = \frac{a^3}{2} \left(2\epsilon M_{\text{Pl}}^2 \left(\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2} \right) + \dot{\mathcal{F}}^2 - \frac{(\partial\mathcal{F})^2}{a^2} - \underbrace{m_s^2 \mathcal{F}^2}_{\text{mixing}} + 4\dot{\sigma}\eta_{\perp}\mathcal{F}\dot{\zeta} \right)$$

$$m_s^2 = V_{;ss} - H^2 \eta_{\perp}^2 + \epsilon R_{\text{fs}} H^2 M_{\text{Pl}}^2$$

Mixing via the bending

Hessian of the potential

Bending of the trajectory

Field-space curvature

$$\frac{\partial^2 \chi}{a^2} = \epsilon \dot{\zeta} + \frac{\dot{\sigma}}{M_{\text{Pl}}^2} \eta_{\perp} \mathcal{F}$$

GENERALIZING MALDACENA'S CALCULATION

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$$\mathcal{L}_{\text{Maldacena}}^{(3)}(\zeta, \chi) = a^3 M_{\text{Pl}}^2 \left[\epsilon(\epsilon - \eta) \dot{\zeta}^2 \zeta + \epsilon(\epsilon + \eta) \zeta \frac{(\partial \zeta)^2}{a^2} + \left(\frac{\epsilon}{2} - 2 \right) \frac{1}{a^4} (\partial \zeta) (\partial \chi) \partial^2 \chi + \frac{\epsilon}{4a^4} \partial^2 \zeta (\partial \chi)^2 \right]$$

[J. Maldacena 2003]

GENERALIZING MALDACENA'S CALCULATION

USING INTEGRATION BY PARTS AND LINEAR EQUATIONS OF MOTION

Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

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New interactions

Boundary terms:
Total time derivatives
contribute to 3-pt functions

[C. Burrage, R. Ribeiro, D. Seery 2011]

[F. Arroja, T. Tanaka 2011]

NEW INTERACTIONS

$$\lambda_{\perp} = \frac{\dot{\eta}_{\perp}}{H\eta_{\perp}} \quad ; \quad \mu_s = \frac{\dot{m}_s}{Hm_s}$$

$$\frac{\partial^2 \chi}{a^2} = \epsilon \dot{\zeta} + \frac{\dot{\sigma}}{M_{\text{Pl}}^2} \eta_{\perp} \mathcal{F}$$

$$\begin{aligned} \mathcal{L}_{\text{new}}^{(3)}(\zeta, \mathcal{F}) = & \frac{1}{2} m_s^2 \zeta \mathcal{F} \left((\epsilon + \mu_s) \mathcal{F} + (2\epsilon - \eta - 2\lambda_{\perp}) \frac{2\dot{\sigma}\eta_{\perp}}{m_s^2} \dot{\zeta} \right) + \frac{\dot{\sigma}\eta_{\perp}}{a^2 H} \mathcal{F} [(\partial\zeta)^2 - \dot{\zeta}^2] \\ & - \frac{1}{H} (H^2 \eta_{\perp}^2 - \epsilon H^2 M_{\text{Pl}}^2 R_{fs}) \dot{\zeta} \mathcal{F}^2 - \frac{1}{6} (V_{;sss} - 2\dot{\sigma} H \eta_{\perp} R_{fs} + \epsilon H^2 M_{\text{Pl}}^2 R_{fs,s}) \mathcal{F}^3 \\ & + \frac{1}{2} \epsilon \zeta \left(\dot{\mathcal{F}}^2 + \frac{(\partial\mathcal{F})^2}{a^2} \right) - \frac{1}{a^2} \dot{\mathcal{F}} (\partial\mathcal{F}) (\partial\chi) \end{aligned}$$

Check: ζ is well massless at any order as it should
(Weinberg adiabatic mode)

$$\mathcal{D} = \frac{M_{\text{Pl}}^2}{2} \frac{d}{dt} [\dots]$$

[Garcia-Saenz, Pinol, Renaux-Petel]
J. High Energ. Phys. **2020**, 73 (2020)

INTEGRATING OUT HEAVY ENTROPIC FLUCTUATIONS

AN EFFECTIVE THEORY FOR THE OBSERVABLE CURVATURE PERTURBATION

$$S[\zeta, \mathcal{F}] \xrightarrow{\mathcal{F}_{\text{heavy}}(\zeta)} S_{\text{EFT}}[\zeta] = S[\zeta, \mathcal{F}_{\text{heavy}}(\zeta)]$$

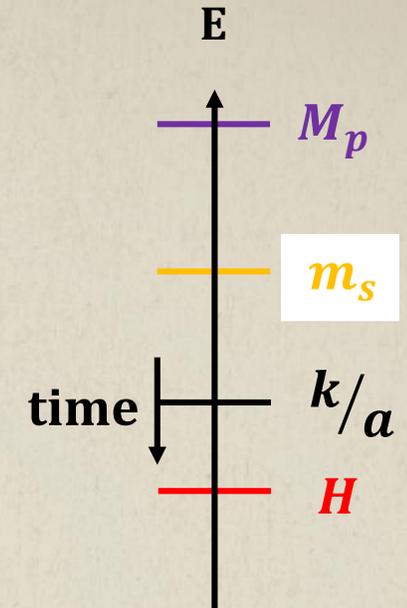
A HIERARCHY OF SCALES

WHEN ENTROPIC FLUCTUATIONS ARE HEAVY

➤ Equation of motion for \mathcal{F} :

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \left(m_s^2 + \frac{k^2}{a^2}\right)\mathcal{F} = 2\dot{\sigma}\eta_{\perp}\dot{\zeta}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

**Integrate out the heavy
perturbation**

*Like in the Fermi theory:
Integrate out the heavy W, Z bosons and
consider contact interactions for fermions*

A HIERARCHY OF SCALES

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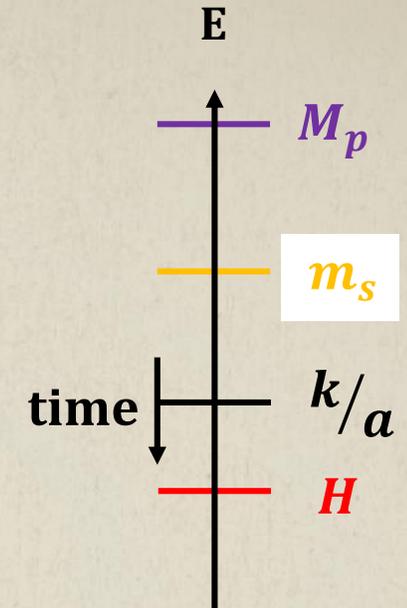
$$\cancel{\ddot{\mathcal{F}}} + 3H\cancel{\dot{\mathcal{F}}} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}} \right) \mathcal{F} = 2\dot{\sigma}\eta_{\perp}\dot{\zeta}$$

When \mathcal{F} is heavy

$$\mathcal{F}_{\text{heavy}}^{\text{LO}} = \frac{2\dot{\sigma}\eta_{\perp}}{m_s^2} \dot{\zeta}$$

$$\omega^2, \omega H, \frac{k^2}{a^2} \ll m_s^2$$

Hierarchy of scales



Energy of the "experiment"

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**Integrate out the heavy
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A HIERARCHY OF SCALES

THE QUADRATIC EFFECTIVE ACTION

➤ Equation of motion for \mathcal{F} :

$$\cancel{\ddot{\zeta}} + 3\cancel{H}\dot{\zeta} + \left(m_s^2 + \cancel{\frac{k^2}{a^2}} \right) \mathcal{F} = 2\dot{\sigma}\eta_{\perp}\dot{\zeta}$$

When \mathcal{F} is heavy

$$\mathcal{F}_{\text{heavy}}^{\text{LO}} = \frac{2\dot{\sigma}\eta_{\perp}}{m_s^2} \dot{\zeta}$$



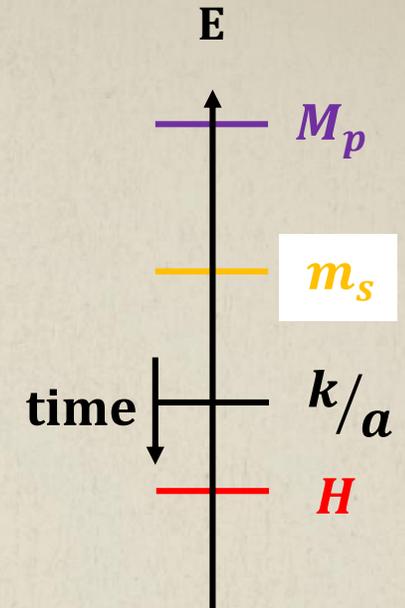
Effective single-field action for the curvature perturbation

$$S_2^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \epsilon \left(\frac{\zeta'^2}{c_s^2} - (\partial_i \zeta)^2 \right)$$

With a speed of sound c_s :

$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_{\perp}^2}{m_s^2}$$

Hierarchy of scales



Energy of the "experiment"

$$H \ll m_s$$

Integrate out the heavy perturbation

*Like in the Fermi theory:
Integrate out the heavy W, Z bosons and
consider contact interactions for fermions*

THE CUBIC EFFECTIVE ACTION

FULL RESULT

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{l} \frac{g_1}{\mathcal{H}} \zeta'^3 + \\ g_2 \zeta'^2 \zeta + \\ g_3 c_s^2 \zeta (\partial_i \zeta)^2 + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1 \right) A \\ g_2 = \epsilon - \eta + 2s \\ \\ g_3 = \epsilon + \eta \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \\ g_4 = \frac{-2\epsilon}{c_s^2} \left(1 - \frac{\epsilon}{4} \right) \\ g_5 = \frac{\epsilon^2}{4c_s^2} \end{array} \right.$$

The only new parameter is A ,
and depends on the UV physics

THE CUBIC EFFECTIVE ACTION

RECOVERING CANONICAL SINGLE-FIELD LIMIT

$$c_s^2 \rightarrow 1$$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{c_s^2}$$

~~The only new parameter is A,
and depends on the UV physics~~

$$\left(\begin{array}{l} \cancel{\frac{g_1}{\mathcal{H}} \zeta'^3} + \\ g_2 \zeta'^2 \zeta + \\ g_3 \zeta (\partial_i \zeta)^2 + \\ \cancel{\frac{\tilde{g}_3}{\mathcal{H}} (\partial_i \zeta)^2} + \\ g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta + \\ g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2 \end{array} \right)$$

Maldacena's result:
Non-Gaussianities $\sim \mathcal{O}(\epsilon, \eta)$

with $\left\{ \begin{array}{l} g_2 = \epsilon - \eta \\ g_3 = \epsilon + \eta \\ g_4 = -2\epsilon \left(1 - \frac{\epsilon}{4} \right) \\ g_5 = \frac{\epsilon^2}{4} \end{array} \right.$

THE CUBIC EFFECTIVE ACTION

RECOVERING THE EFT OF INFLATION

$$\epsilon, \eta, s \rightarrow 0$$

Slow-varying result:

$$\text{Non-Gaussianities} \sim \frac{1}{c_s^2} - 1$$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{c_s^2} \left(\begin{array}{l} \frac{g_1}{\mathcal{H}} \zeta'^3 + \\ \cancel{g_2 \zeta'^2 \zeta} + \\ \cancel{g_3 c_s^2 \zeta (\partial_i \zeta)^2} + \\ \frac{\tilde{g}_3 c_s^2}{\mathcal{H}} \zeta' (\partial_i \zeta)^2 + \\ \cancel{g_4 \zeta' \partial_i \partial^{-2} \zeta' \partial_i \zeta} + \\ \cancel{g_5 \partial^2 \zeta (\partial_i \partial^{-2} \zeta')^2} \end{array} \right) \text{ with } \left\{ \begin{array}{l} g_1 = \left(\frac{1}{c_s^2} - 1 \right) A \\ \tilde{g}_3 = \frac{1}{c_s^2} - 1 \end{array} \right.$$

The only new parameter is A,
and depends on the UV physics

THE EFT OF INFLATION

REVISITED...

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right)$$

$$\text{with } A = \underbrace{-\frac{1}{2} (1 + c_s^2)} + \dots$$

Previously known

THE EFT OF INFLATION

REVISITED...

$$\text{Bending radius of the trajectory: } \kappa = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right)$$

$$\text{with } A = -\frac{1}{2} (1 + c_s^2) - \underbrace{\frac{1}{6} (1 - c_s^2) \frac{\kappa V_{;SSS}}{m_s^2}}_{\text{3rd derivative of the potential (expected)}} + \dots$$

**3rd derivative of the potential
(expected)**

Self-coupling of entropic fluctuations

THE EFT OF INFLATION

REVISITED...

Bending radius of the trajectory: $\kappa = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right)$$

$$\text{with } A = -\frac{1}{2}(1 + c_s^2) - \frac{1}{6}(1 - c_s^2) \frac{\kappa V_{;SSS}}{m_s^2} + \underbrace{\frac{2}{3}(1 + 2c_s^2) \frac{\epsilon R_{\text{fs}} H^2 M_{\text{Pl}}^2}{m_s^2}}_{\text{Scalar curvature of the field space}} + \dots$$

Scalar curvature of the field space

THE EFT OF INFLATION

REVISITED...

Bending radius of the trajectory: $\kappa = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right)$$

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Derivative of the
scalar curvature

THE EFT OF INFLATION

REVISITED...

Bending radius of the trajectory: $\kappa = \frac{\sqrt{2\epsilon}M_p}{\eta_\perp}$

$$S_3^{\text{EFT}}[\zeta] = \int d\tau d^3\vec{x} a^2 M_{\text{Pl}}^2 \frac{\epsilon}{\mathcal{H}} \left(\frac{1}{c_s^2} - 1 \right) \left(\zeta' (\partial_i \zeta)^2 + \frac{A}{c_s^2} \zeta'^3 \right)$$

$$\text{with } A = \underbrace{-\frac{1}{2}(1 + c_s^2)}_{\text{Previously known}} + \underbrace{\frac{2}{3}(1 + 2c_s^2) \frac{\epsilon R_{\text{fs}} H^2 M_{\text{Pl}}^2}{m_s^2}}_{\text{3rd derivative of the potential}} - \frac{1}{6}(1 - c_s^2) \left(\underbrace{\frac{\kappa V_{;SSS}}{m_s^2}}_{\text{Derivative of the scalar curvature}} + \underbrace{\frac{\kappa \epsilon H^2 M_{\text{Pl}}^2 R_{\text{fs},s}}{m_s^2}}_{\text{Derivative of the scalar curvature}} \right)$$

Previously known

3rd derivative of the potential

Scalar curvature of the field space

Derivative of the scalar curvature

Then you can use the result of the EFToI:

Equilateral shape with:

[Garcia-Saenz, Pinol, Renaux-Petel]

J. High Energ. Phys. **2020**, 73 (2020)

THE EFT OF INFLATION

REVISITED...

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Previously known

3rd derivative of the potential

Scalar curvature of the field space

Derivative of the scalar curvature

$$f_{\text{NL}}^{\text{eq}} \simeq \frac{1}{18} \left(\frac{1}{c_s^2} - 1 \right) \left(A - \frac{17}{4} \right)$$

All contributions matter, none is a priori negligible

THE EFT OF INFLATION

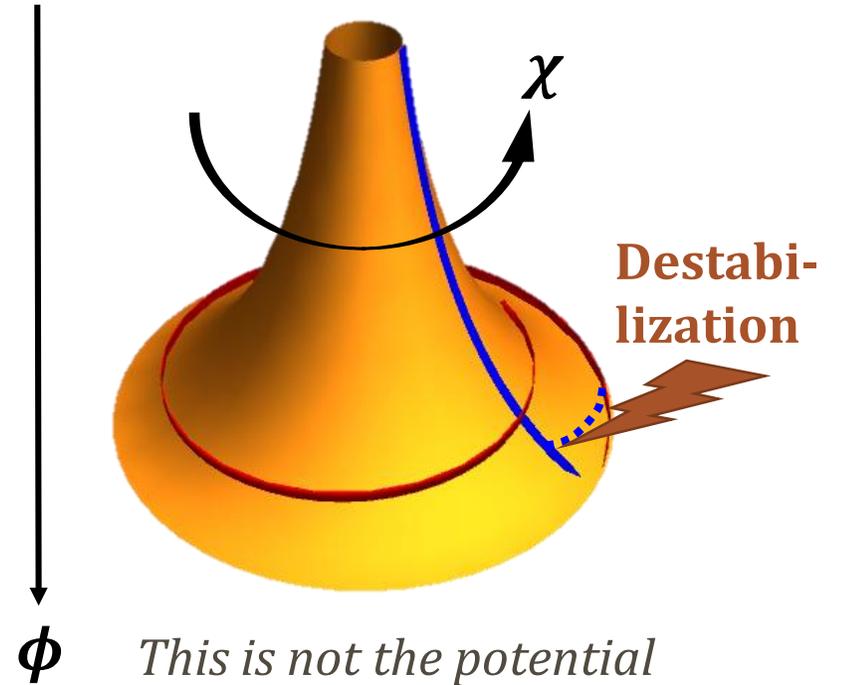
HYPERINFLATION

An exotic case where conditions to integrate out are fulfilled

[Fumagalli, Garcia-Saenz, Pinol,
Renaux-Petel, Ronayne 2019]

Phys. Rev. Lett. 123, 201302

- Embedding of the hyperbolic plane in 3D
- Radial trajectory
- Hyperinflation trajectory



Hyperbolic field space

$$R_{\text{fs}} = -\frac{4}{M^2}, \quad M \ll M_p$$

THE EFT OF INFLATION

HYPERINFLATION

[Fumagalli, Garcia-Saenz, Pinol,
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An exotic case where conditions to integrate out are fulfilled

➤ Our new formula enables to **compute**

$$c_s^2 \simeq -1$$

$$A \simeq -0.33$$

0 without the geometric $\propto R_{fs}$ contribution



THE EFT OF INFLATION

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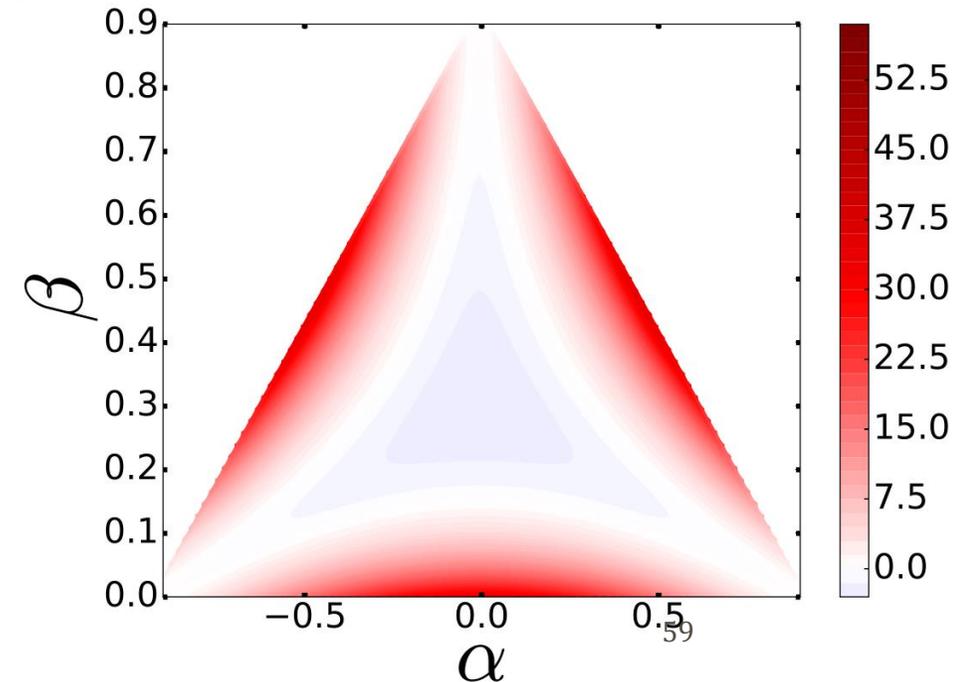
$$c_s^2 \simeq -1$$

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- Analytical prediction for the whole shape of the bispectrum:

vs. numerical resolution for the full multified model?



THE EFT OF INFLATION

HYPERINFLATION

[Fumagalli, Garcia-Saenz, Pinol, Renaux-Petel, Ronayne 2019]

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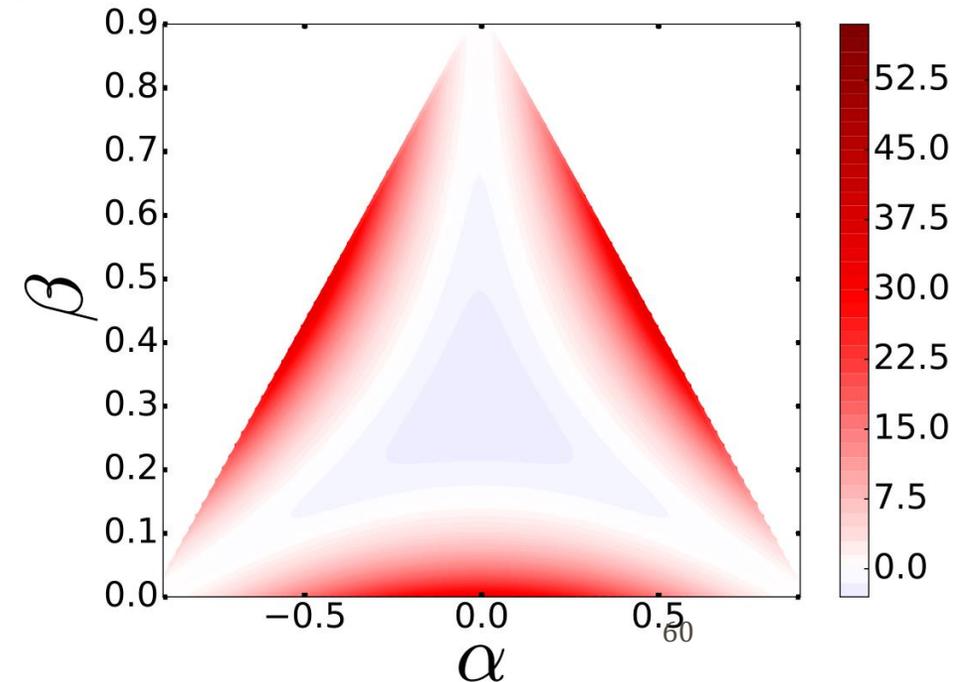
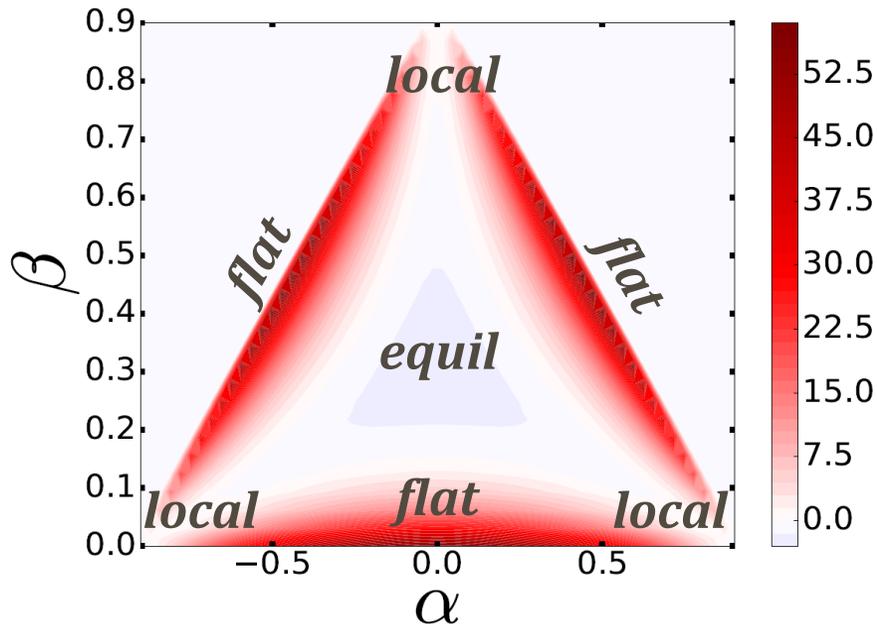
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- Analytical prediction for the whole shape of the bispectrum:



THE EFT OF INFLATION

HYPERINFLATION

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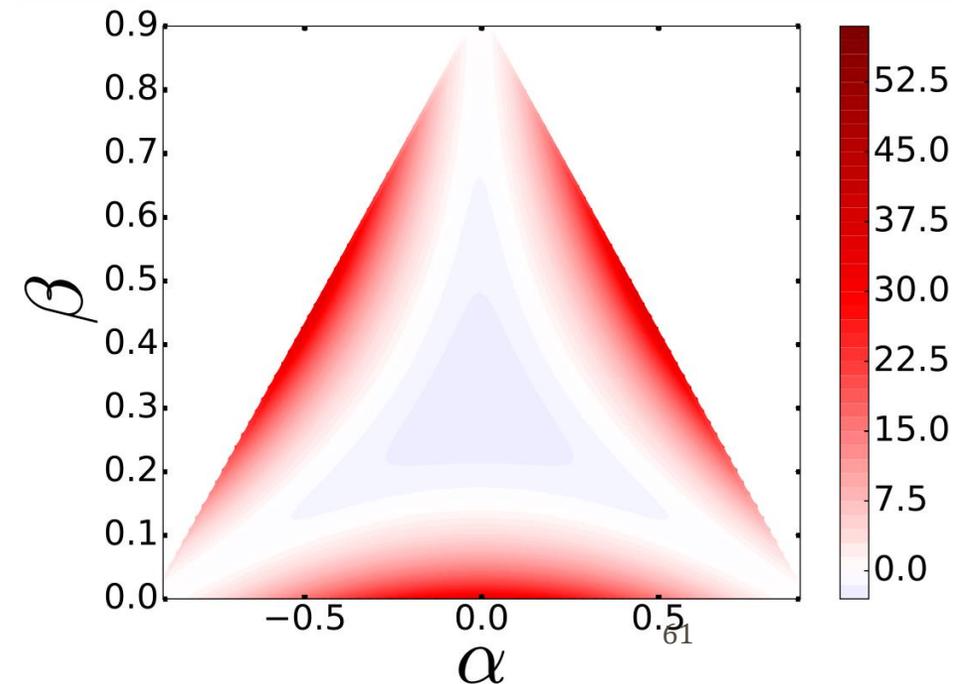
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FROM 2 FIELDS TO N FIELDS

[Pinol 2020]

TOWARDS A MORE GENERAL UNDERSTANDING

J. Cosm. & Astro. Phys. 04(2021)048

Resulting Lagrangian, after $O(40)$ integrations by parts and $O(10)$ uses of equations of motion:

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FROM 2 FIELDS TO N FIELDS

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$$\mathcal{L}^{(2)}(\zeta, \mathcal{F}^\alpha) = \frac{a^3}{2} \left(2\epsilon M_{\text{Pl}}^2 \left(\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2} \right) + 4\sqrt{2}\epsilon M_{\text{Pl}} \omega_1 \mathcal{F}^1 \dot{\zeta} + \dot{\mathcal{F}}^{\alpha 2} - \frac{(\partial\mathcal{F}^\alpha)^2}{a^2} - \underbrace{m_{\alpha\beta}^2 \mathcal{F}^\alpha \mathcal{F}^\beta}_{\text{Purely entropic mixing via the torsion matrix}} + \Omega_{\alpha\beta} \dot{\mathcal{F}}^\alpha \mathcal{F}^\beta \right)$$

Adiabatic-entropic mixing via the bending

$$m_{\alpha\beta}^2 = V_{;\alpha\beta} - \delta_{\alpha 1} \delta_{\beta 1} \omega_1^2 + (\Omega^2)_{\alpha\beta} + 2\epsilon H^2 M_{\text{Pl}}^2 R_{\alpha\sigma\beta\sigma}$$

Covariant hessian of the potential

Bending / torsion of the trajectory

Field-space curvature

FROM 2 FIELDS TO N FIELDS

TOWARDS A MORE GENERAL UNDERSTANDING

[Pinol 2020]

J. Cosm. & Astro. Phys. 04(2021)048

$$\begin{aligned}
 \mathcal{L}^{(3)} = & M_p^2 a^3 \left[\epsilon(\epsilon - \eta) \dot{\zeta}^2 \zeta + \epsilon(\epsilon + \eta) \zeta \frac{(\partial\zeta)^2}{a^2} + \left(\frac{\epsilon}{2} - 2\right) \frac{1}{a^4} (\partial\zeta)(\partial\chi) \partial^2 \chi + \frac{\epsilon}{4a^4} \partial^2 \zeta (\partial\chi)^2 \right] \\
 & + a^3 \left\{ \sqrt{2\epsilon} \omega_1 M_{\text{Pl}} \left[\frac{\mathcal{F}^1}{H} \left(\frac{(\partial\zeta)^2}{a^2} - \dot{\zeta}^2 - \dot{\zeta} \zeta H (\eta + 2u_1) \right) + 2 \frac{\Omega_{1\alpha}}{H} \dot{\zeta} \zeta \mathcal{F}^\alpha \right] \right. \\
 & + \left[\frac{\epsilon}{2} m_{\alpha\beta}^2 + \frac{(\dot{m}_{\alpha\beta}^2)}{2H} + \Omega_{\gamma\beta} \left(\epsilon \Omega^\gamma{}_\alpha + \frac{\dot{\Omega}^\gamma{}_\alpha}{H} - \frac{m_{\gamma\alpha}^2}{H} \right) \right] \zeta \mathcal{F}^\alpha \mathcal{F}^\beta + \epsilon \Omega_{\alpha\beta} \zeta \dot{\mathcal{F}}^\alpha \mathcal{F}^\beta \\
 & + (2\epsilon H^2 M_{\text{Pl}}^2 R_{\alpha\sigma\beta\sigma} - \omega_1^2 \delta_{\alpha 1} \delta_{\beta 1}) \frac{\dot{\zeta}}{H} \mathcal{F}^\alpha \mathcal{F}^\beta + \frac{1}{2} \epsilon \zeta \left((\dot{\mathcal{F}}^\alpha)^2 + \frac{(\partial\mathcal{F}^\alpha)^2}{a^2} \right) \\
 & - \frac{1}{a^2} (\partial\mathcal{F}^\alpha) (\partial\chi) (\dot{\mathcal{F}}^\alpha + \Omega_{\alpha\beta} \mathcal{F}^\beta) + \frac{2}{3} \sqrt{2\epsilon} H M_{\text{Pl}} R_{\alpha\beta\gamma\sigma} \dot{\mathcal{F}}^\alpha \mathcal{F}^\beta \mathcal{F}^\gamma \\
 & \left. - \frac{1}{6} \left(V_{;\alpha\beta\gamma} - 4\sqrt{2\epsilon} H M_{\text{Pl}} \left(\omega_1 \delta_{\alpha 1} R_{\beta\sigma\gamma\sigma} + \Omega^\delta{}_\alpha R_{\delta\beta\gamma\sigma} \right) + 2\epsilon H^2 M_{\text{Pl}}^2 R_{\alpha\sigma\beta\sigma;\gamma} \right) \mathcal{F}^\alpha \mathcal{F}^\beta \mathcal{F}^\gamma \right\} \\
 & + \mathcal{D},
 \end{aligned} \tag{3.10}$$

$\mathcal{L}_{\text{new}}^{(3)}(\zeta, \mathcal{F}^\alpha)$

APPLICATIONS

[Pinol 2020]

J. Cosm. & Astro. Phys. 04(2021)048

- Single-field EFT from N_{field} inflation as a realistic UV completion:

$$\frac{1}{c_s^2} = 1 + 4\omega_1^2 (m^{-2})_{11}$$

*Bending to deviate
from $c_s^2 = 1$*

*Inverse squared
mass matrix*

$$A = -\frac{1}{2}(1 + c_s^2) + \frac{4}{3}(1 + 2c_s^2)\epsilon H^2 M_p^2 (m^{-2})_{11} R_{m\sigma m\sigma} \\ - \frac{\kappa}{6}(1 - c_s^2) (m^{-2})_{11} \left[V_{;mmm} + 2\epsilon H^2 M_p^2 R_{m\sigma m\sigma;m} \right. \\ \left. + 4\sqrt{2\epsilon} H M_p \left(\Omega_m^\alpha + \frac{1}{(m^{-2})_{11}} \frac{d(m^{-2})_{11}^\alpha}{dt} \right) R_{m\alpha m\sigma} \right],$$

$$\underline{\text{EX:}} R_{m\sigma m\sigma} = \frac{(m^{-2})_{1\alpha} (m^{-2})_{1\beta}}{[(m^{-2})_{11}]^2} R_{\alpha\sigma\beta\sigma}$$

The whole mass matrix of entropic fields matters! (not just m_{11}^2)

The whole geometry of the non-linear sigma model matters! (not just the Ricci scalar R_{fs})

- Cosmological collider with $N_{\text{field}}-1$ entropic fields: what is the bispectrum in the squeezed limit?

The cosmic spectroscopy (beyond single-field EFT)

INFLATIONARY FLAVOR AND MASS BASES

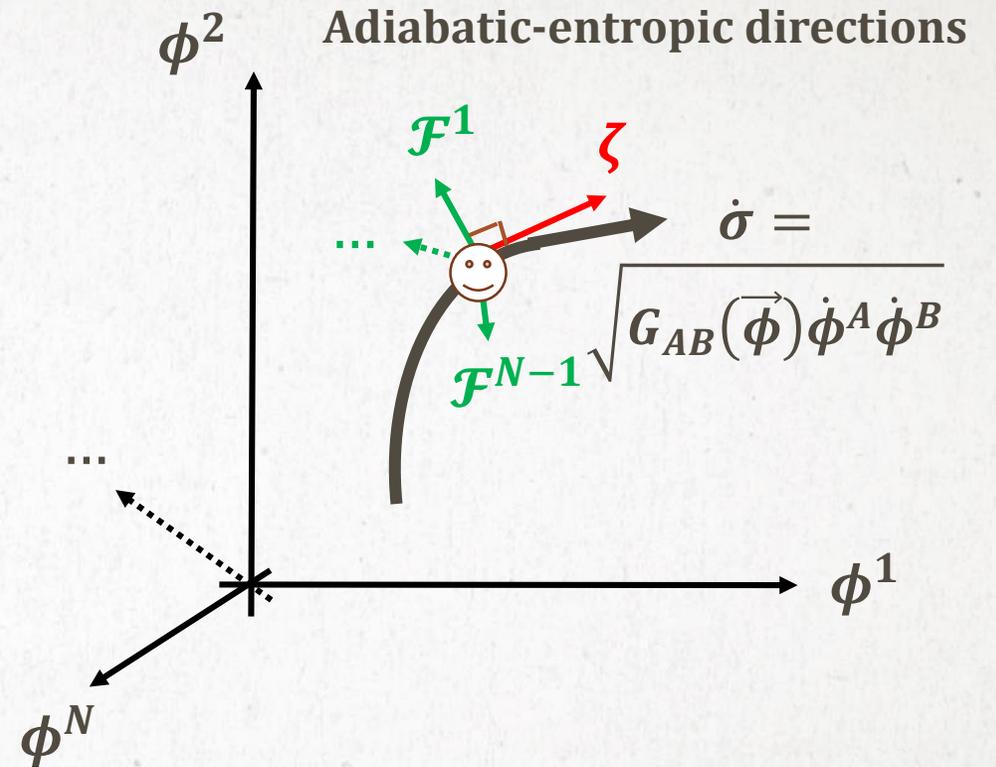
[Lucas Pinol 2020]

J. Cosm. & Astro. Phys. 04(2021)048

Quadratic action for the extra fluctuations:

(Also the cubic action is computed)

- ζ The curvature perturbation
- $\mathcal{F}^1, \dots, \mathcal{F}^{N-1}$ The entropic/isocurvature perturbations



INFLATIONARY FLAVOR AND MASS BASES

[Lucas Pinol 2020]

J. Cosm. & Astro. Phys. 04(2021)048

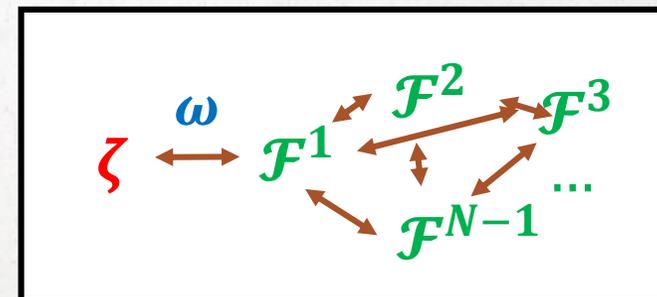
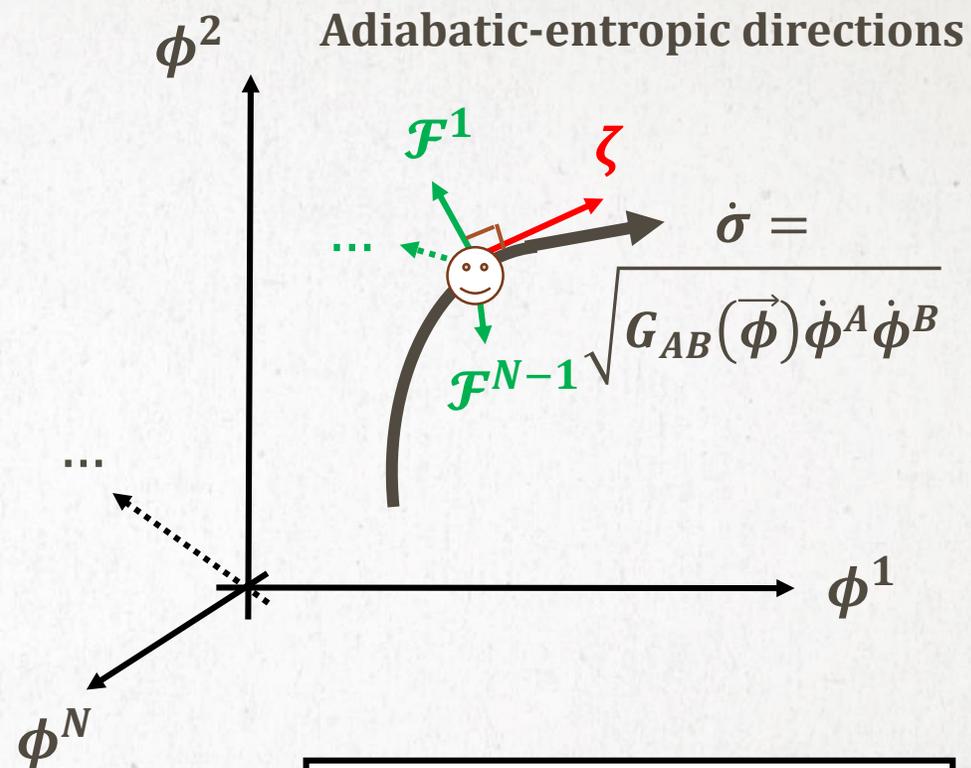
Quadratic action for the extra fluctuations:

$$\mathcal{L}_{\text{flavor}}^{(2)} = \frac{a^3}{2} \left[\delta_{\alpha\beta} \left(\dot{\mathcal{F}}^\alpha \dot{\mathcal{F}}^\beta - \frac{\partial \mathcal{F}^\alpha \partial \mathcal{F}^\beta}{a^2} \right) - \underline{M_{\alpha\beta}^2} \mathcal{F}^\alpha \mathcal{F}^\beta \right] + 4 \sqrt{2\epsilon} M_{\text{Pl}} \underline{\omega \delta_{\alpha 1}} \mathcal{F}^\alpha \dot{\zeta}$$

- Non-trivial mass matrix mixing
- Only the first extra field \mathcal{F}^1 is coupled to ζ :
portal field + sterile sector

Flavor basis: the one in which interactions are specified

Diagonalization: $M_{\alpha\beta} = (OmO^T)_{\alpha\beta}$ and $\mathcal{F}^\alpha = O^\alpha_i \sigma^i$



INFLATIONARY FLAVOR AND MASS BASES

[Lucas Pinol, Aoki, Renaux-Petel, Yamaguchi 2021]

ArXiv:2112.05710

Quadratic action for the extra fluctuations:

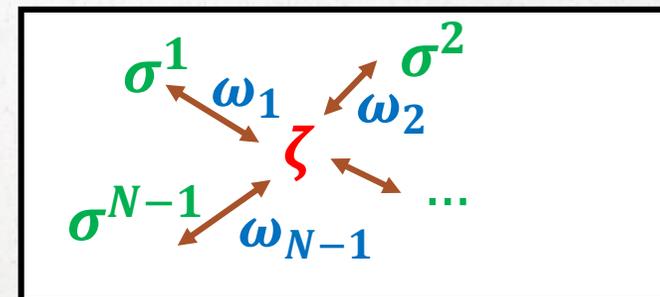
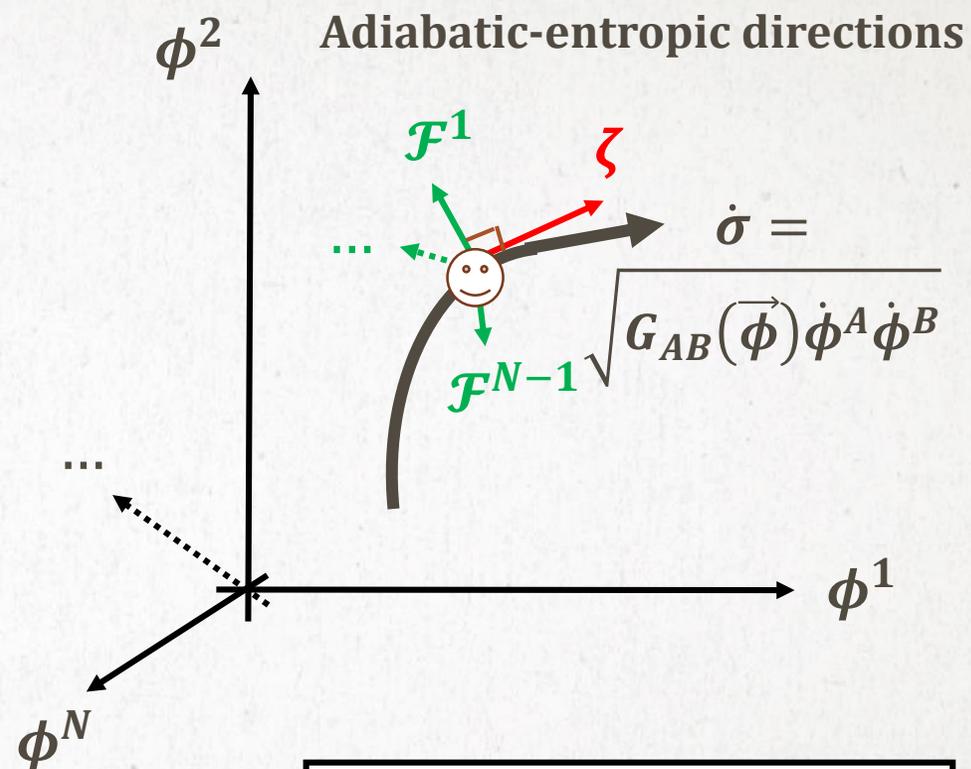
$$\mathcal{L}_{\text{mass}}^{(2)} = \frac{a^3}{2} \left[\delta_{ij} \left(\dot{\sigma}^i \dot{\sigma}^j - \frac{\partial \sigma^i \partial \sigma^j}{a^2} \right) - \sum_i \underline{m_i^2 \sigma_i^2} \right] + 4 \sqrt{2\epsilon} M_{\text{Pl}} \underline{\omega O^1}_i \sigma^i \dot{\zeta}$$

- Well-defined masses
- All mass eigenstates are coupled to ζ with:

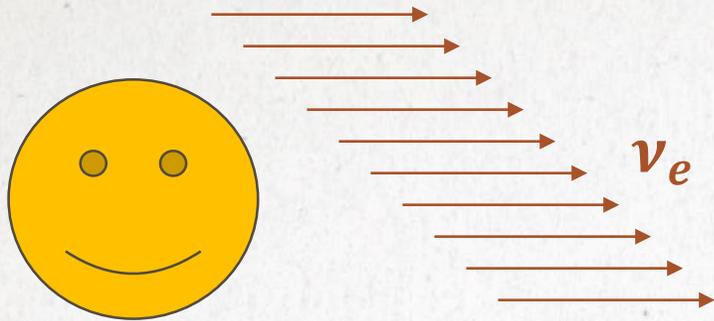
$$\omega_i = \omega O^1_i \text{ with } \mathcal{F}^1 = O^1_i \sigma^i$$

Mass basis: the one in which masses are specified

Diagonalization: $M_{\alpha\beta} = (OmO^T)_{\alpha\beta}$ and $\mathcal{F}^\alpha = O^\alpha_i \sigma^i$



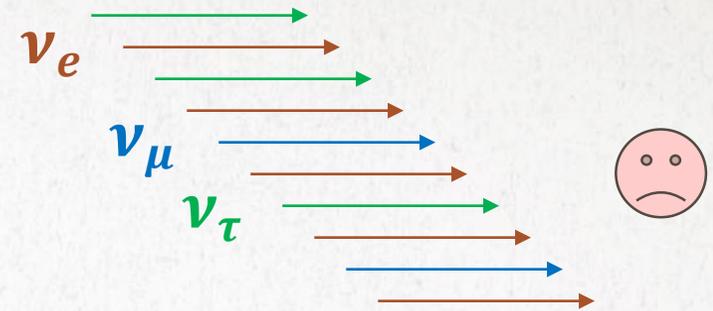
ANALOGY WITH NEUTRINO OSCILLATIONS



This is the Sun

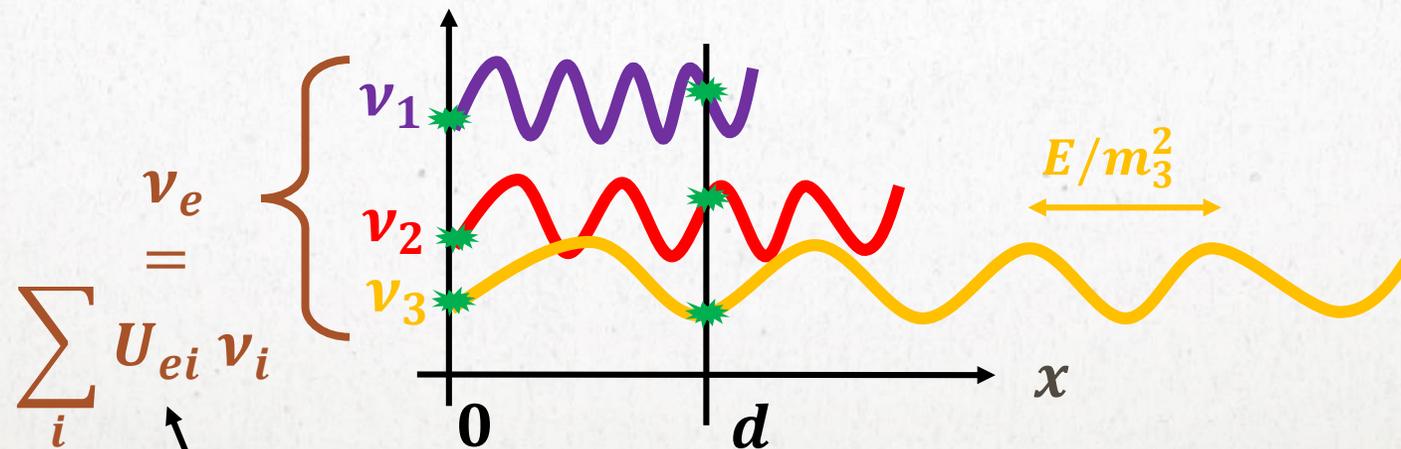
It is emitting electronic neutrinos*

NO INTERACTIONS



This is me

I am seeing many less electronic neutrinos



$$\nu_e = \sum_i U_{ei} \nu_i$$

Entries of the PMNS matrix: mixing angles

PMNS stands for Pontecorvo-Maki-Nakagawa-Sakata:
try to pronounce it ten times in a row

*also some ν_τ from MSW

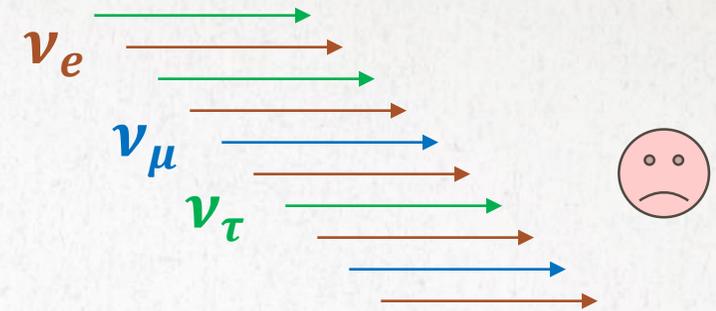
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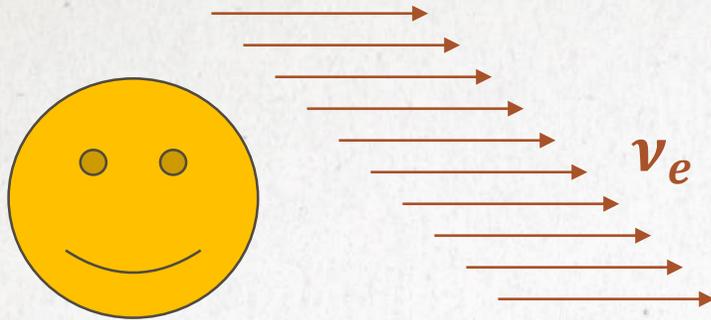
I am seeing many less electronic neutrinos

For us, \mathcal{F}^α are the flavor eigenstates and σ_i the free fields: the mass eigenstates.

In particular: $\mathcal{F}^1 = \sum_i O^1_i \sigma_i$ with $O^1_i = [\cos(\theta_{12})\cos(\theta_{13}), \sin(\theta_{12})\cos(\theta_{13}), \sin(\theta_{13})]_i$, (3)

Mixing angles if $N_{\text{flavor}} = 3$

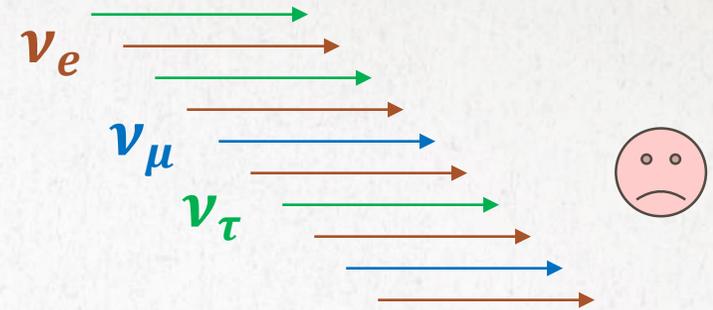
ANALOGY WITH NEUTRINO OSCILLATIONS



This is the Sun

It is emitting electronic neutrinos

NO INTERACTIONS



This is me

I am seeing many less electronic neutrinos

What process equivalent to the missing solar neutrinos may hint towards inflationary flavor oscillations?

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

Single-field result:

$$f_{\text{NL}}^{\text{squeezed}} \propto 1 - n_s \ll 1$$

consistency relation

[Maldacena 2003]

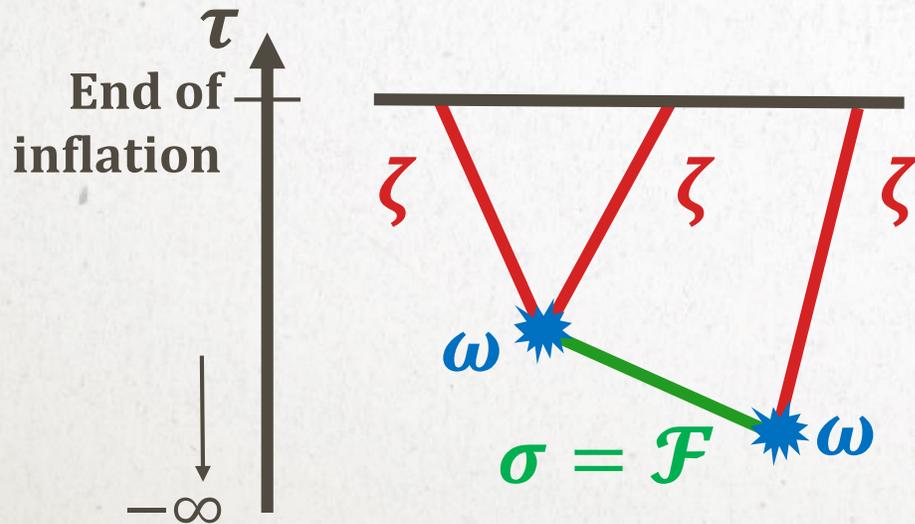
Two-field result:

Usual curvature perturbation ζ + one heavy field $\sigma = \mathcal{F}$ (no flavor oscillation)

[Chen, Wang 2009]

[Noumi, Yamaguchi, Yokoyama 2013]

[Arkani-Hamed, Maldacena 2015]



$$f_{\text{NL}}^{\text{squeezed}} \sim \left(\frac{\omega}{H}\right)^2 e^{-\pi\mu} \cos \left[\mu \log \left(\frac{k_l}{k_s} \right) + \varphi(\mu) \right]$$

Oscillatory pattern: massive particle

$$\mu = \sqrt{\frac{m_\sigma^2}{H^2} - \frac{9}{4}} \text{ the reduced mass}$$

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

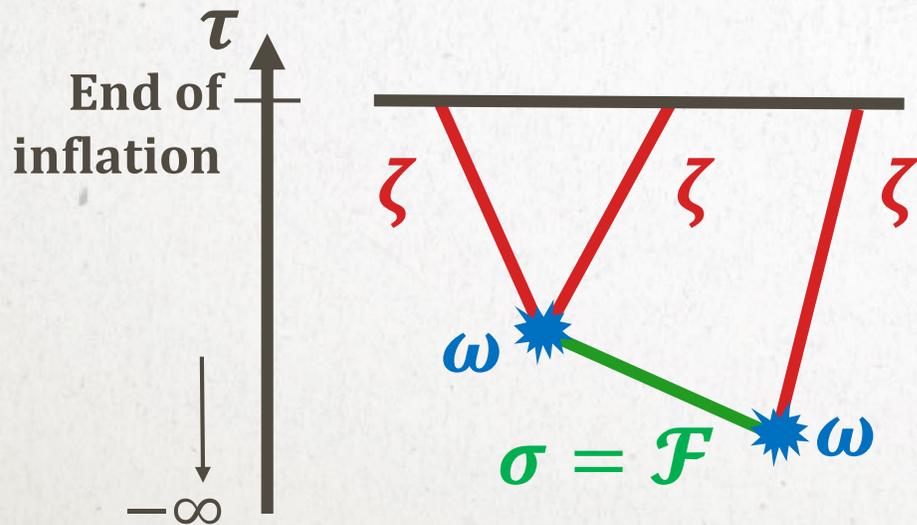
Single-field result:

$$f_{\text{NL}}^{\text{squeezed}} \propto 1 - n_s \ll 1$$

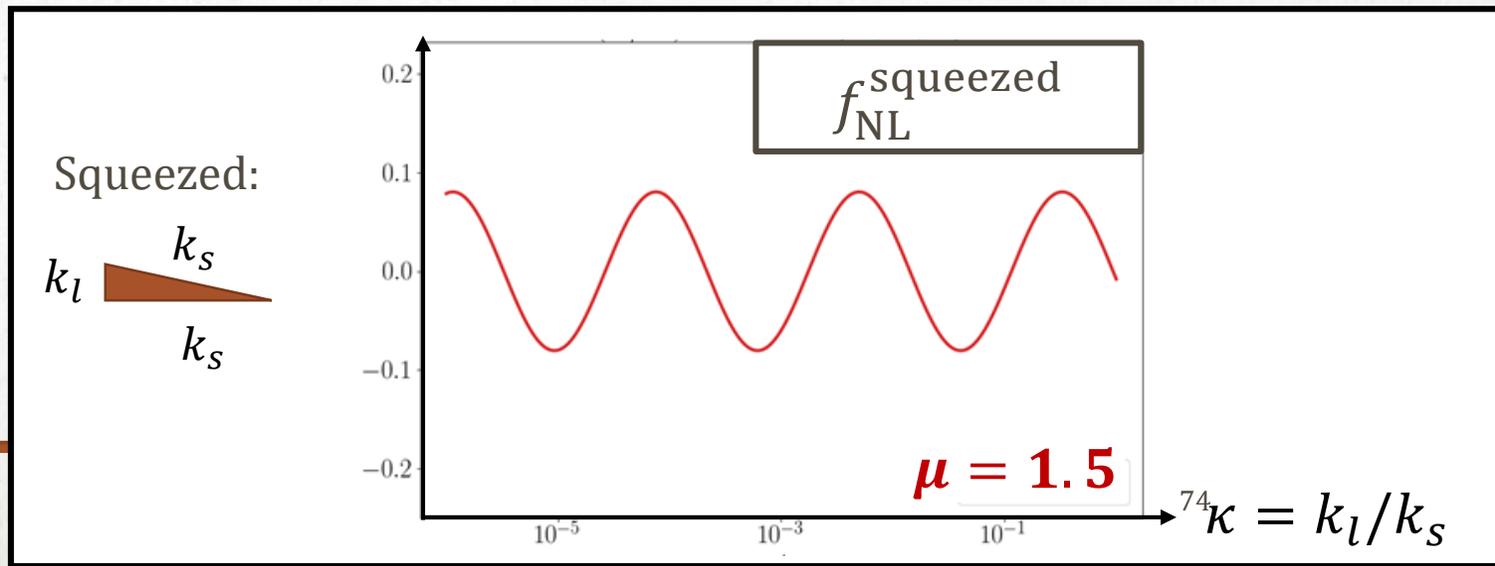
consistency relation

Two-field result:

Usual curvature perturbation ζ + one heavy field $\sigma = \mathcal{F}$ (no flavor oscillation)



Super-Hubble oscillations of a massive scalar field



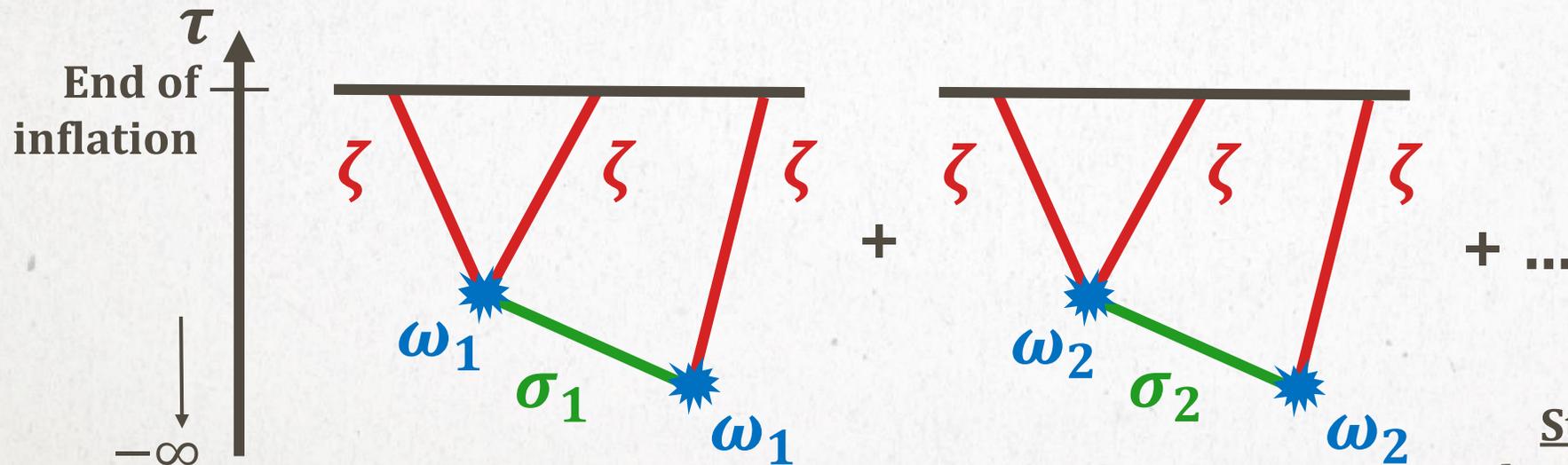
BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

[Lucas Pinol, Aoki, Renaux-Petel, Yamaguchi 2021]

ArXiv:2112.05710

N-field result: ... this work!



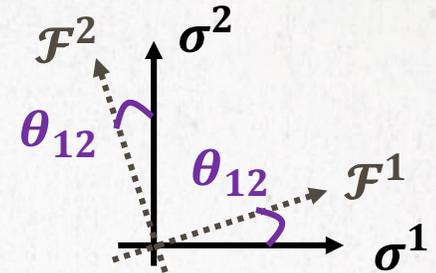
Strength of interactions depend on the mixing angles

$$\omega_i = O^1_i \times \omega$$

BISPECTRUM IN MULTIFIELD INFLATION

- ❖ Three fields: ζ and 2 flavors ($\mathcal{F}^1, \mathcal{F}^2$)

↳ Only one **mixing angle** θ_{12}

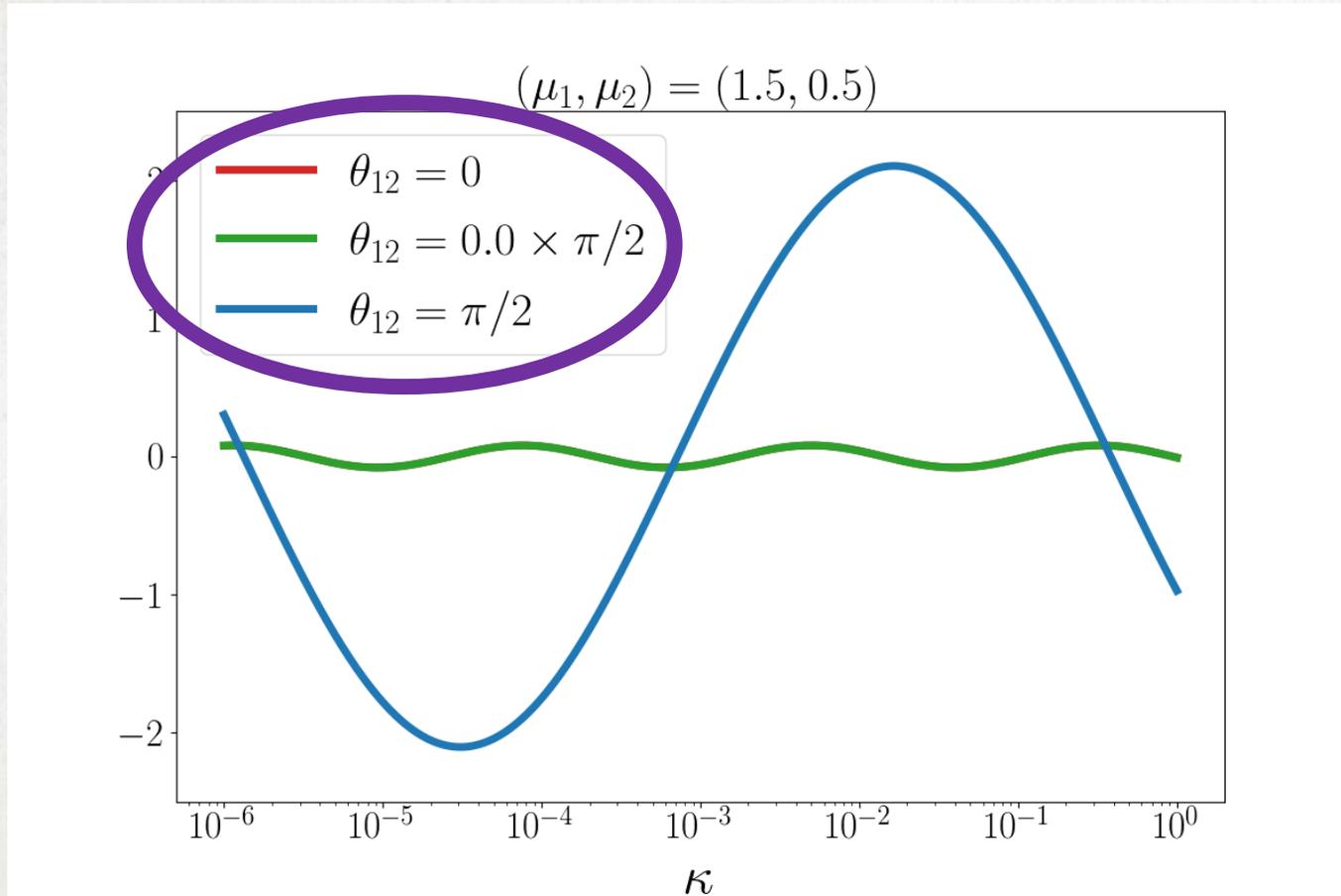


- ❖ If $\theta_{12} \in \{0, \pi/2\}$: no mixing

- ❖ If $0 < \theta_{12} < \pi/2$: mixing

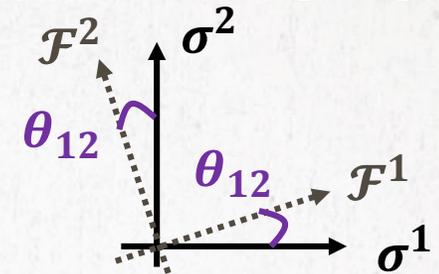
BISPECTRUM IN MULTIFIELD INFLATION

For heavy mass eigenstates σ_1, σ_2



❖ Three fields: ζ and 2 flavors ($\mathcal{F}^1, \mathcal{F}^2$)

↳ Only one **mixing angle** θ_{12}



❖ If $\theta_{12} \in \{0, \pi/2\}$: no mixing

↳ **Oscillations** with frequency $\mu_{1,2}$

❖ If $0 < \theta_{12} < \pi/2$: mixing

↳ **Modulated oscillations** with frequencies $\frac{\mu_1 \pm \mu_2}{2}$

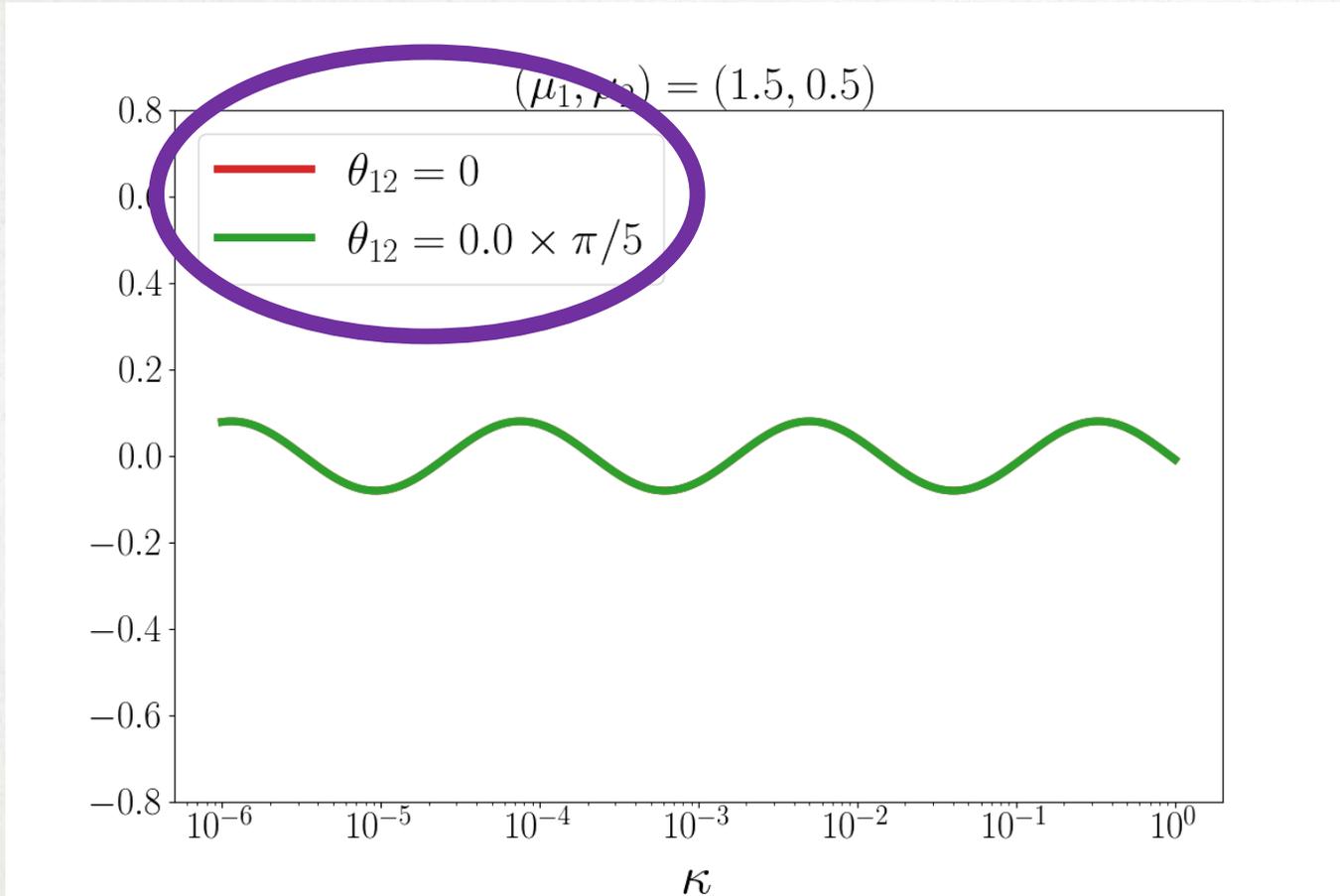
[Lucas Pinol, Aoki, Renaux-Petel, Yamaguchi 2021]

ArXiv:2112.05710

See the paper for all cases⁷³ and any N

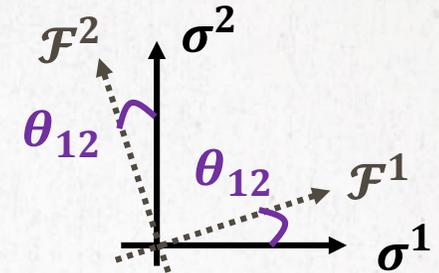
BISPECTRUM IN MULTIFIELD INFLATION

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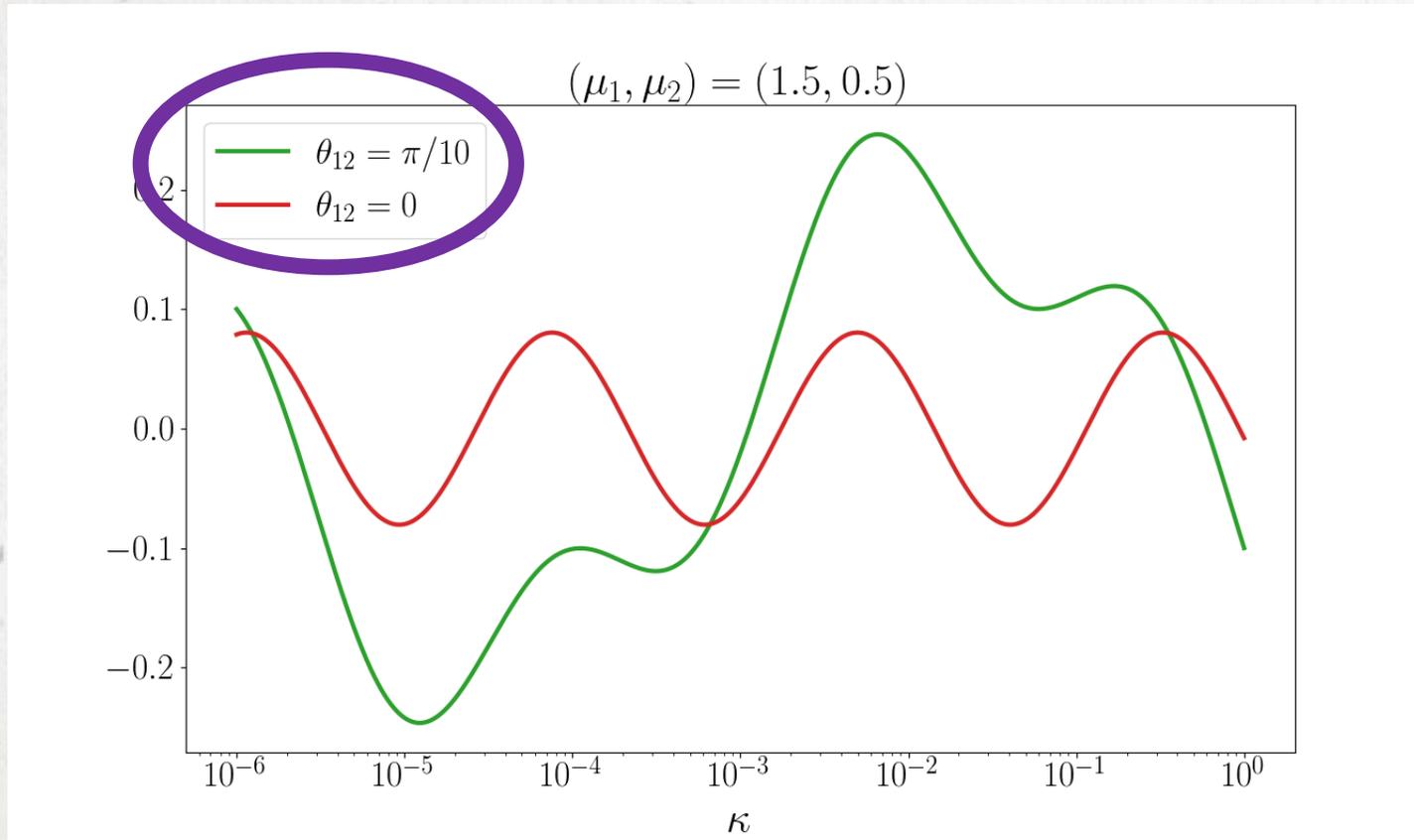
[Lucas Pinol, Aoki, Renaux-Petel, Yamaguchi 2021]

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See the paper for all cases and any N

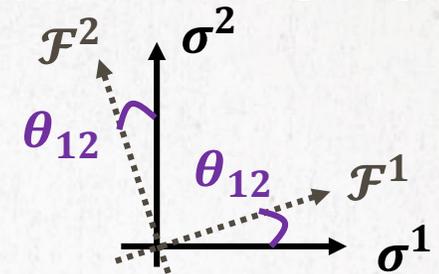
BISPECTRUM IN MULTIFIELD INFLATION

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[Lucas Pinol, Aoki, Renaux-Petel, Yamaguchi 2021]

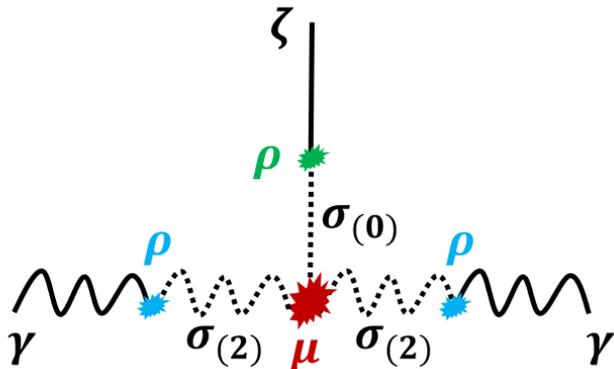
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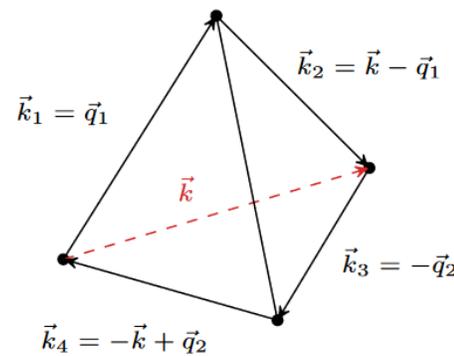
III. PNG AND GW: AN INTERTWINED STORY

GW anisotropies of primordial origin

Scalar-trispectrum-induced GW: a no-go theorem



*Mixed bispectrum
(scalar-tensor-tensor)
inducing GW anisotropies*



*Tetrahedron shape
for the scalar trispectrum
inducing GW*

OTHER KINDS OF PNG

- Higher-order correlation functions:

$$\text{SSSS} \quad \left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \right\rangle_c = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \times T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$$

Trispectrum



etc.

- Tensor and mixed scalar-tensor PNG

$$\text{SST} \quad \left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \gamma_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times B_{\zeta\zeta\gamma}(k_1, k_2, k_3)$$

$$\text{STT} \quad \left\langle \zeta_{\vec{k}_1} \gamma_{\vec{k}_2} \gamma_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times B_{\zeta\gamma\gamma}(k_1, k_2, k_3)$$

$$\text{TTT} \quad \left\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \gamma_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \times B_{\gamma\gamma\gamma}(k_1, k_2, k_3)$$

All these correlators are observable and contain information about high-energy physics and inflation

OTHER KINDS OF PNG: CONSTRAINTS

Bounds at CMB scales

- Higher-order correlation functions:

$$\text{SSSS} \xrightarrow{\text{squeezed}} g_{\text{NL}} = (-5.8 \pm 6.5)10^4 \text{ [Planck 2018]}$$

$$\text{SSSS} \xrightarrow{\text{collapsed}} \tau_{\text{NL}} = 400 \pm 1300 \text{ [Marzouk, Lewis, Carron 2022]}$$

from theory...

$$\tau_{\text{NL}} \geq \left(\frac{6}{5} f_{\text{NL}}^{\text{loc}}\right)^2 \text{ (= in single-field only)}$$

- Tensor and mixed scalar-tensor PNG

$$\text{SST} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\zeta\zeta\gamma} = -48 \pm 28 \text{ [Shiraishi, Liguori, Fergusson 2017]}$$

$$\text{STT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\zeta\gamma\gamma} = ???$$

$$\text{TTT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\gamma\gamma\gamma} = 220 \pm 170 \text{ [WMAP 2013]}$$

[Suyama, Yamaguchi 2007]

[Smith, LoVerde, Zaldarriaga 2011]

... and nothing else...

OTHER KINDS OF PNG: CONSTRAINTS

- Higher-order correlation functions:

$$\text{SSSS} \xrightarrow{\text{squeezed}} g_{\text{NL}} = (-5.8 \pm 6.5)10^4 \quad [\text{Planck 2018}]$$

$$\text{SSSS} \xrightarrow{\text{collapsed}} \tau_{\text{NL}} = 400 \pm 1300 \quad [\text{Marzouk, Lewis, Carron 2022}]$$

- Tensor and mixed scalar-tensor PNG

$$\text{SST} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\zeta\zeta\gamma} = -48 \pm 28 \quad [\text{Shiraishi, Liguori, Fergusson 2017}]$$

$$\text{STT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\zeta\gamma\gamma} = ???$$

$$\text{T TT} \xrightarrow{\text{squeezed}} f_{\text{NL,loc}}^{\gamma\gamma\gamma} = 220 \pm 170 \quad [\text{WMAP 2013}]$$

END OF THIS TALK

GW anisotropies of primordial origin

THE STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

Primordial gravitational waves constitute a key prediction from inflation!

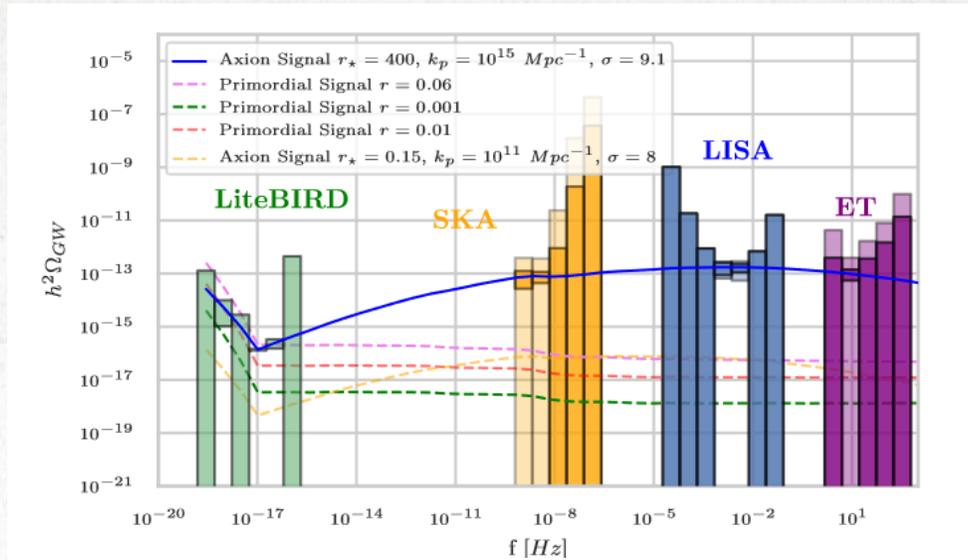
But...

Many sources! Astrophysical, cosmological... How to disentangle them?

DISTINCTIVE FEATURES OF THE SGWB

Frequency profile

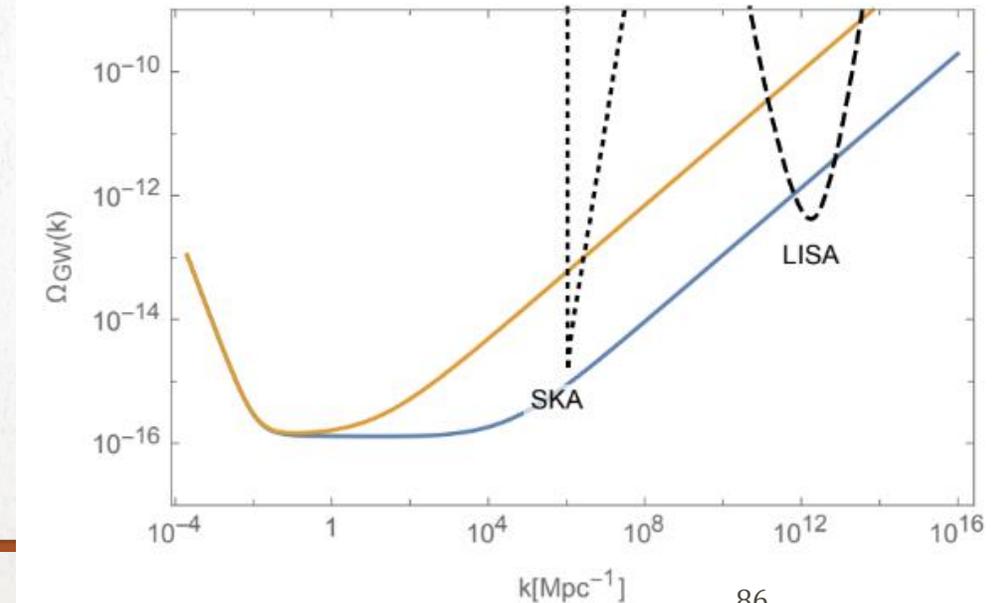
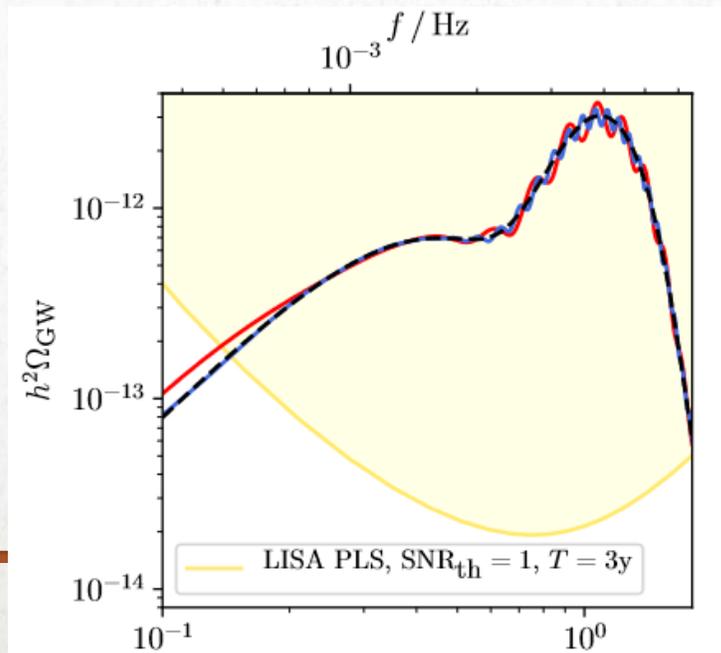
$$\bar{\Omega}_{GW}(f) = \Omega_0 \left(\frac{f}{f_*} \right)^{n_{GW}(f)}$$



Having access to several orders of magnitude in frequency can help

[Auclair *et al.*, LISA CWG 2022]

[Many many works, sorry for not showing yours]



DISTINCTIVE FEATURES OF THE SGWB

Chirality

Often in the context of a Cherns-Simon term

- Gauge fields: $g(\chi)F^{a\mu\nu}\tilde{F}_{\mu\nu}^a \in \mathcal{L}$

[Anber, Sorbo 2010, 2011]

[Barnaby, Peloso 2011]

[Dimastrogiovanni, Peloso 2013]

[Adshead, Martinec, Wyman 2013]

[Dimastrogiovanni, Fasiello, Fujita 2016]

[Watanabe, Komatsu 2020]

- Beyond GR: $g(\chi)R^{\mu\nu}\tilde{R}_{\mu\nu} \in \mathcal{L}$

[Bartolo, Orlando 2017, 2018]

Unstable polarisation that sources **chiral** GWs:

$$\gamma_L \gg \gamma_R$$

Chirality $\chi = \frac{|P_\gamma^L - P_\gamma^R|}{P_\gamma^{\text{tot}}}$ can be measured

Also the possibility of other modes in the GWs

DISTINCTIVE FEATURES OF THE SGWB

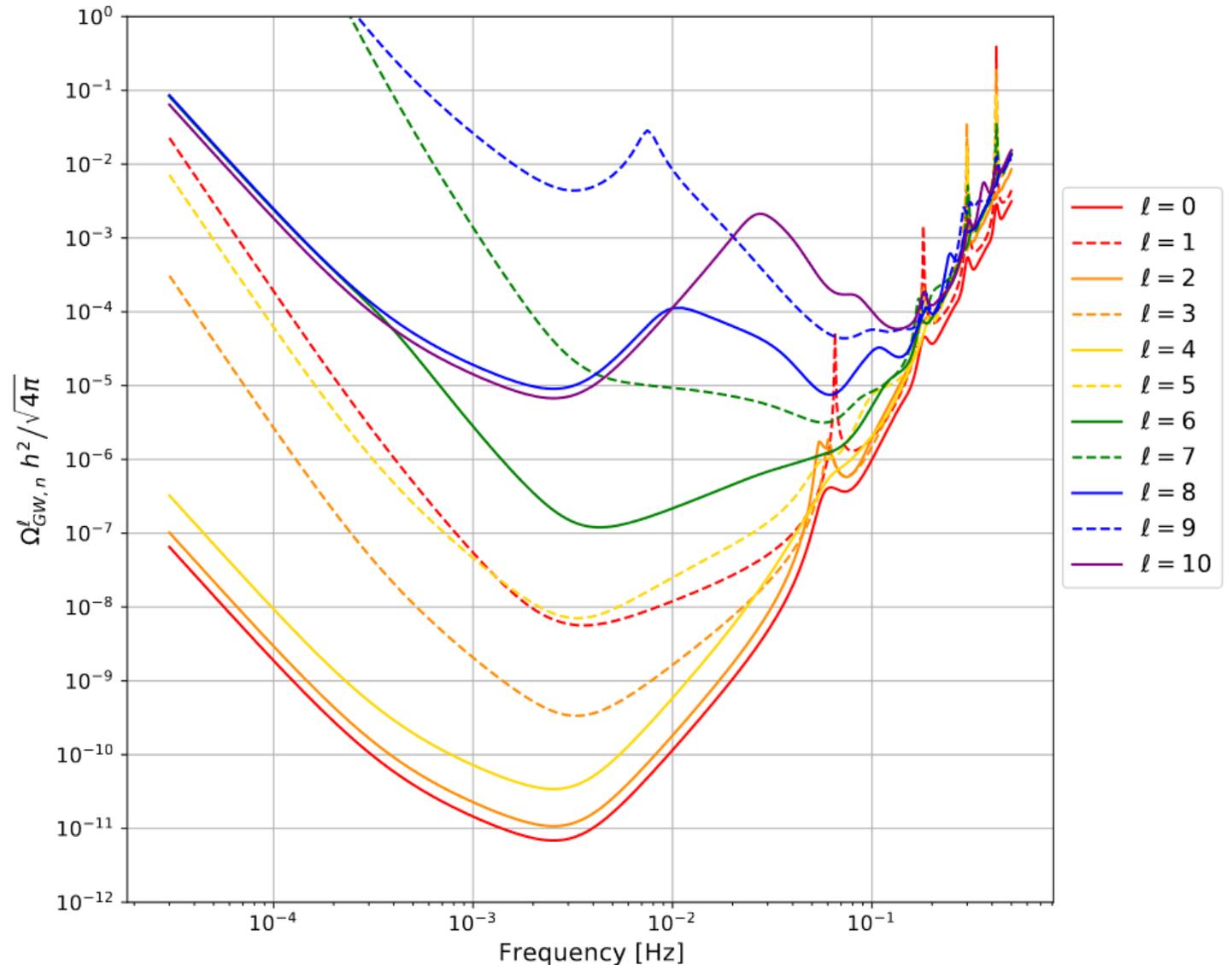
Anisotropies:

$$\Omega_{GW}(f, \hat{n}) = \bar{\Omega}_{GW}(f)(1 + \delta_{GW}(f, \hat{n}))$$

$$a_{\ell, m} = \int d\Omega Y_{\ell, m}(\hat{n}) \delta_{GW}(\hat{n})$$

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_m a_{\ell, m}^* a_{\ell, m}$$

Different sources give different anisotropies



SEVERAL SOURCES OF ANISOTROPIES

- GWs signal from astrophysical sources expected to be anisotropic
[Cusin *et al.* 2017, 2018, 2019]
[Bertacca *et al.* 2019]
[Bellomo *et al.* 2021]
- Cosmological background propagates through structures → anisotropic
[Alba, Maldacena 2015]
[Contaldi *et al.* 2016]
[Bartolo *et al.* 2018, 2019] *These anisotropies inherit a non-Gaussian statistics from propagation*
[Domcke, Jinno, Rubira 2020]
- Primordial NGs also induce anisotropies:
[Jeong, Kamionkowski 2012] *Anisotropies of the LSS from the same effect*
[Brahma, Nelson, Chandra 2013]
[Dimastrogiovanni *et al.* 2014, 2015, 2021]

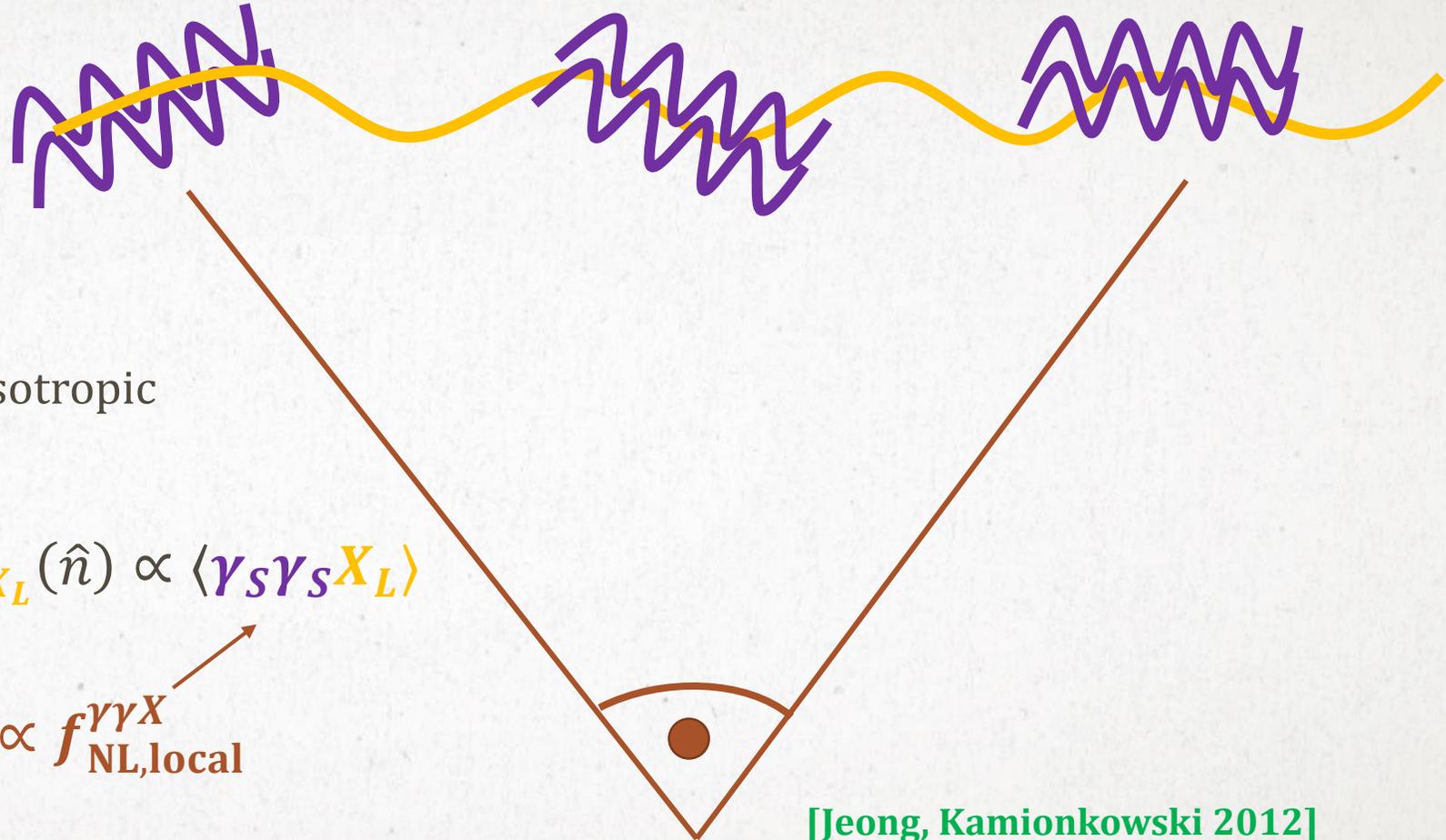
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- **Primordial NGs also induce anisotropies:**
[Jeong, Kamionkowski 2012]
[Brahma, Nelson, Chandra 2013]
[Dimastrogiovanni *et al.* 2014, 2015, 2021]

PNG-INDUCED ANISOTROPIES IN THE SGWB

- The idea:

Consider the modulation of **two short modes** by a **long one**:
seen from far away the signal is anisotropic



$$\langle \gamma_s \gamma_s \rangle \rightarrow \delta_{\text{GW}}(\hat{n}, f_s) \propto \langle \gamma_s \gamma_s \rangle_{X_L}(\hat{n}) \propto \langle \gamma_s \gamma_s X_L \rangle$$

$$\frac{\Omega_{\text{GW}}(\hat{n}, f)}{\bar{\Omega}_{\text{GW}}(f)} - 1$$

$$\propto f_{\text{NL,local}}^{\gamma\gamma X}$$

[Jeong, Kamionkowski 2012]

Here γ_s is a tensor (anisotropies of the SGWB) but first introduced for scalars (anisotropies of LSS)

Also X_L can be ζ_L (modulation by a soft scalar mode) or γ_L (modulation by a soft tensor mode)

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

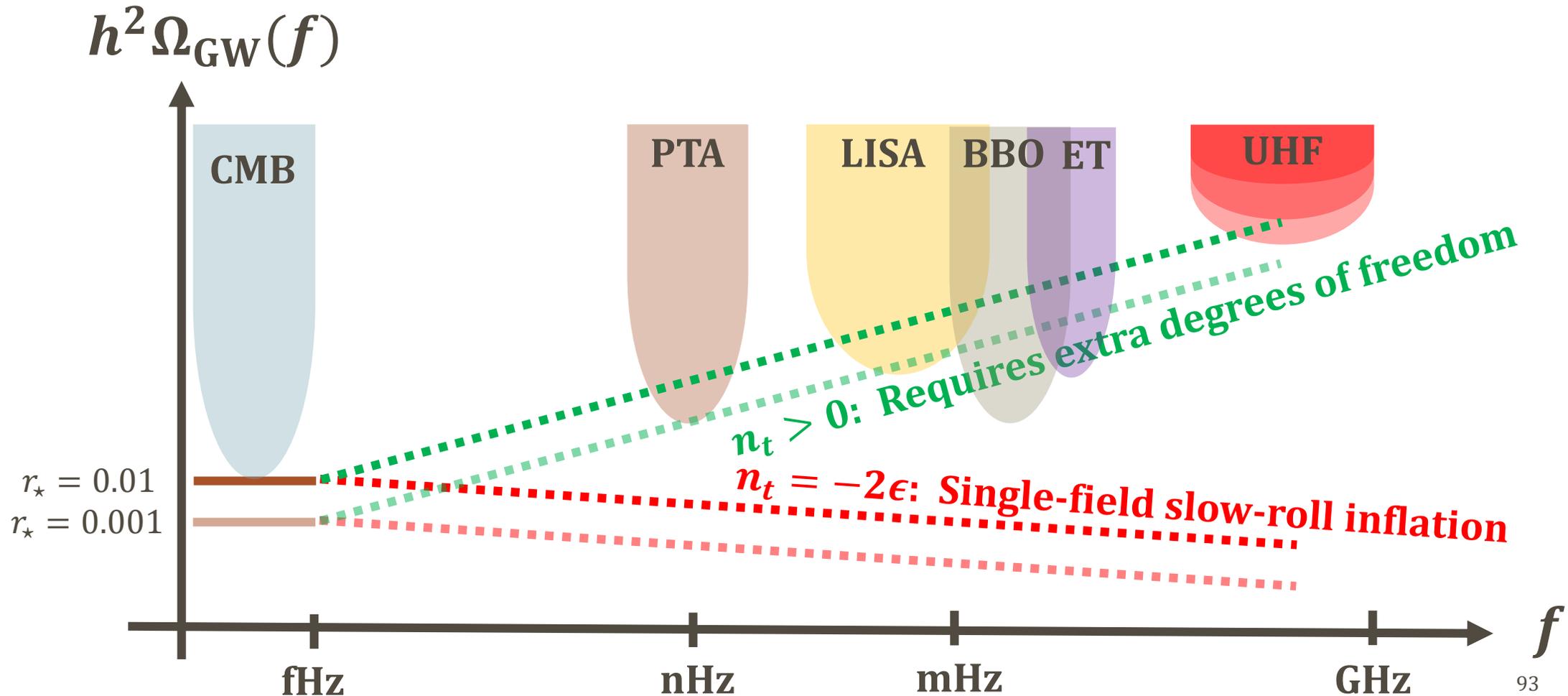
of primordial origin!

- Having an observable monopole signal

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

- Having an observable monopole signal → smaller scales, requires a blue tilt: $n_t > 0$



INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

- Having an observable monopole signal: $n_t > 0$
- Having large STT or TTT bispectra in the (ultra) squeezed limit

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

➤ Having an observable monopole signal: $n_t > 0$

➤ Having large STT or TTT bispectra in the (ultra) squeezed limit: $f_{\text{NL,sq}}^{\zeta\gamma\gamma}, f_{\text{NL,sq}}^{\gamma\gamma\gamma} \gg 1$

$$\left\langle \gamma_{\vec{k}_1}^{\lambda_1} \gamma_{\vec{k}_2}^{\lambda_2} \right\rangle_{\gamma_{\vec{q}_L}} = \sum_{\lambda_3} \int_{|\vec{q}| < q_L} d^3 q \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q}) \gamma_{\mathbf{q}}^{*\lambda_3} \frac{B_{\gamma}^{\lambda_1 \lambda_2 \lambda_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q})}{P_{\gamma}^{\lambda_3}(q)}$$


“heuristic” formula of the literature

[Ricciardone, Tasinato 2017]

[Dimastrogiovanni, Fasiello, Tasinato 2019]

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

- Having an observable monopole signal: $n_t > 0$
- Having large STT or TTT bispectra in the (ultra) squeezed limit: $f_{\text{NL,sq}}^{\zeta\gamma\gamma}, f_{\text{NL,sq}}^{\gamma\gamma\gamma} \gg 1$
- That this squeezed limit is not due to spurious residual gauge artifacts

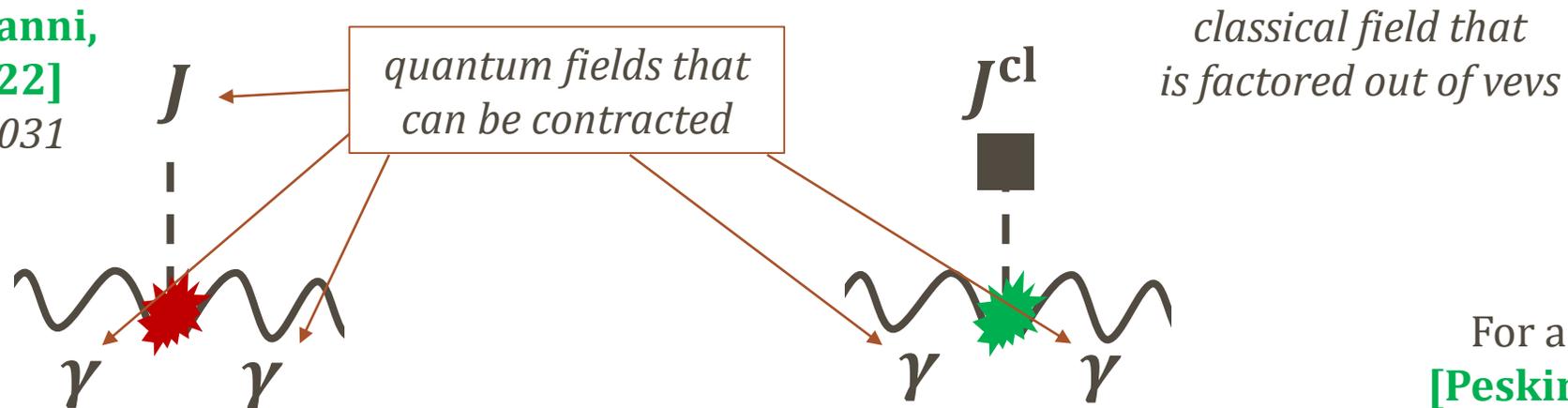
INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

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This work: ❖ Go beyond the heuristic approach and **compute** the two-point function with a classical source

[Dimastrogiovanni,
Fasiello, LP 2022]
JCAP 09 (2022) 031



For a QED example see
[Peskin, Schroeder 1995]

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

- Having an observable monopole signal: $n_t > 0$
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[Dimastrogiovanni,
Fasiello, LP 2022]

JCAP 09 (2022) 031

This work: ❖ Go beyond the heuristic approach and **compute** the two-point function with a classical source

$$\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\text{cl}}} \Big|_{|\vec{k}_1 + \vec{k}_2| \ll k_{1,2}} = \int d^3 \vec{q} \delta^{(3)}(\vec{q} + \vec{k}_1 + \vec{k}_2) P_\gamma(k_1) f_{\text{NL,sq}}^{J\gamma\gamma}(\vec{k}_1, \vec{k}_2, \vec{q}) J^{\text{cl}}(\vec{q})$$

Non-diagonal part, $\vec{k}_1 + \vec{k}_2 \neq \vec{0}$, of the 2-pt function does not vanish \rightarrow anisotropies

$J^{\text{cl}}(\vec{q})$ is a statistical quantity \rightarrow so is $\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\text{cl}}} \rightarrow \langle \delta(\hat{n}_1) \delta(\hat{n}_2) \rangle \propto \langle J^{\text{cl}}(\vec{q}) J^{\text{cl}}(\vec{q}') \rangle \neq 0$ ₉₈

INGREDIENTS FOR OBSERVABLE GW ANISOTROPIES

of primordial origin!

➤ Having an observable monopole signal: $n_t > 0$ 

➤ Having large STT or TTT bispectra in the (ultra) squeezed limit: $f_{\text{NL,sq}}^{\zeta\gamma\gamma}, f_{\text{NL,sq}}^{\gamma\gamma\gamma} \gg 1$ 

➤ That this **squeezed limit is not due to spurious residual gauge artifacts** 

[Dimastrogiovanni,
Fasiello, LP 2022]

JCAP 09 (2022) 031

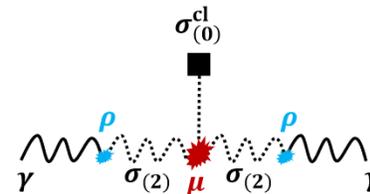
This work: ❖ Go beyond the heuristic approach and compute the two-point function with a classical source

❖ Prove that some already existing inflationary models verify **all 3 requirements above**

Example 1: EFT of spin-2 field

[Bordin *et al.* 2018]

$$\sigma_{ij} = \partial_i \partial_j \sigma^{(0)} + \sigma_{ij}^{(2)}$$



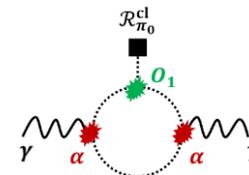
Example 2: Supersolid inflation

[Celoria *et al.* 2021]

Two scalars (ζ_n, R_{π_0})

adiabatic

entropic



$$\delta_{\text{GW}}(\hat{n}) \geq 10\%$$

Scalar-trispectrum-induced GW: a no-go theorem

SCALAR-INDUCED GW

❖ At horizon re-entry in the radiation era:

$$\gamma_k'' + 2\mathcal{H}\gamma_k' + k^2\gamma_k = \mathcal{S}_k$$

Source term including scalar perturbations at quadratic order

$$\propto \int d^3\vec{q} (\dots) \zeta_{\vec{q}} \zeta_{\vec{k}-\vec{q}}$$

SCALAR-INDUCED GW

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- ❖ The tensor two-point function is proportional to the scalar four-point function:

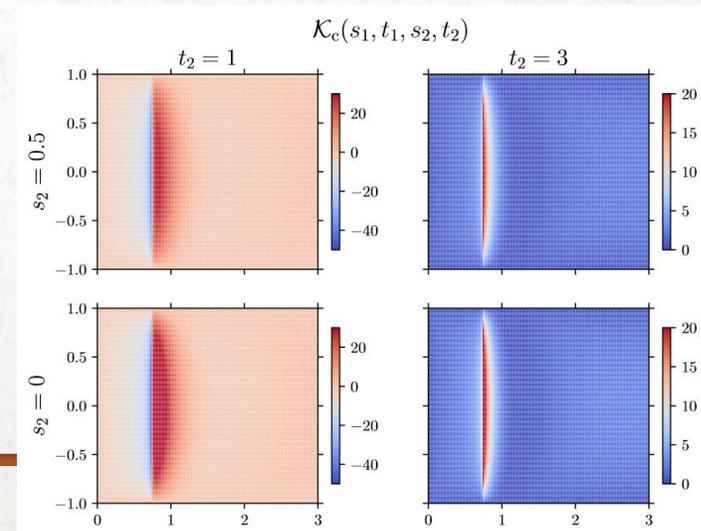
$$P_\gamma(k) = \int d^3\vec{q}_1 \int d^3\vec{q}_2 \mathcal{K}(\vec{q}_1, \vec{q}_2) \times \langle \zeta_{\vec{q}_1} \zeta_{\vec{k}-\vec{q}_1} \zeta_{-\vec{q}_2} \zeta_{-\vec{k}+\vec{q}_2} \rangle$$

general kernel

[Adshead, Lozanov, Weiner 2021]

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv: 2207.14267



we discuss its symmetries etc.

SCALAR-INDUCED GW

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$$P_\gamma(k) = \int d^3\vec{q}_1 \int d^3\vec{q}_2 \mathcal{K}(\vec{q}_1, \vec{q}_2) \times \langle \zeta_{\vec{q}_1} \zeta_{\vec{k}-\vec{q}_1} \zeta_{-\vec{q}_2} \zeta_{-\vec{k}+\vec{q}_2} \rangle$$

Disconnected (Gaussian) piece:

$$(2\pi)^3 \delta^{(3)}(\vec{q}_1 - \vec{q}_2) P_\zeta(q_1) P_\zeta(|\vec{k} - \vec{q}_1|)$$

+ perm.

[Many works]

Connected (non-Gaussian) piece:

$$T_\zeta(\vec{q}_1, \vec{k} - \vec{q}_1, -\vec{q}_2, -\vec{k} + \vec{q}_2)$$

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv: 2207.14267

SCALAR-TRISPECTRUM-INDUCED GW

❖ Only a few recent works working out *some* scalar trispectrum effects:

[Garcia-Bellido, Peloso, Unal 2017]
[Unal 2018]
[Atal, Domenech 2021]
[Adshead, Lozanov, Weiner 2021]

❖ But *all* limited themselves to **local non-linearities**: $\zeta = \zeta_G + f_{\text{NL}}^{\text{loc}} \zeta_G^2$

renormalizes the power spectrum: $P_\zeta = P_{\zeta_G} + 3f_{\text{NL}}^2 (P_{\zeta_G})^2$ induces NGs: $\langle \zeta^4 \rangle_{\text{connected}} \propto f_{\text{NL}}^2 P_{\zeta_G}^3 + \mathcal{O}(f_{\text{NL}}^3)$

... and did not check perturbative control → large effects from NGs

NO-GO THEOREM FOR SCALAR-TRISPECTRUM-INDUCED GW

Lemma. Given real symmetric matrices A and B , with A positive definite, then $C \equiv AB$ is diagonalizable (over the complex numbers) and has real eigenvalues.

This work:

[Garcia-Saenz, LP, Renaux-Petel, Werth 2022]

ArXiv:

2207.14267

Local shapes

“Equilateral” shapes:
interactions from EFTol

“Cosmo. collider” shapes:
exchange of massive and
spinning fields

- ❖ we investigate motivated scalar trispectrum shapes
- ❖ AND we check perturbative control



Shape	$\Omega_{\text{connected}}^{\text{GW}} / \Omega_{\text{disconnected}}^{\text{GW}}$	Perturbativity bound
\mathcal{G}_{NL}	0	...
τ_{NL}	$4 \times \tau_{\text{NL}} \mathcal{P}_{\zeta} \log(kL)$	$\tau_{\text{NL}} \mathcal{P}_{\zeta} \log(kL) \ll 1$
$t_{\text{NL}}^{\zeta^4}, t_{\text{NL}}^{\zeta^2(\partial\zeta)^2}, t_{\text{NL}}^{(\partial\zeta)^4}$	 0 or negligible	...
$t_{\text{NL}}^{[\zeta^3]^2}, t_{\text{NL}}^{[\zeta(\partial\zeta)^2]^2}, t_{\text{NL}}^{\zeta^3 \times \zeta(\partial\zeta)^2}$	 $\mathcal{O}(10^{-1}) \times (H/\Lambda_{\star})^4$ Numerically computed coefficient	$H/\Lambda_{\star} \ll 1$
$\tau_{\text{NL}}^{\text{exchange}}(\Delta, S)$	 $4f(\Delta, S) \times \tau_{\text{NL}} \mathcal{P}_{\zeta} \log(kL)$?

$f(\Delta, S) < 1$

L is the size of the Universe (IR cutoff)

Λ_{\star} is the smallest strong coupling scale

CONCLUSION...

- Primordial NGs contain much more information than a single number $f_{\text{NL}}^{\text{local}}$
- Depending on the mass spectrum, mixing angles and interactions of primordial field content, scalar and tensor PNGs are of different **amplitudes** and **shapes**
- Small-scale ultra-squeezed STT and TTT PNGs survive in the form of induced anisotropies in the SGWB
- The scalar trispectrum sources GWs at horizon re-entry but its relative contribution must remain small
IN SCALE-INVARIANT MODELS

↳ **Warning for scale-dependent models:
compute perturbativity bounds!**

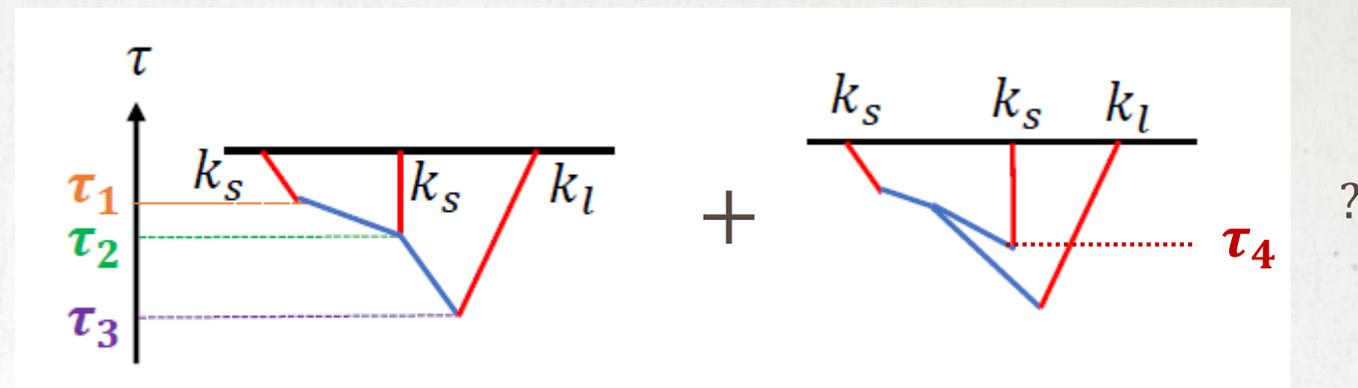


**Formidable opportunity to use
the non-linear Universe as a
particle detector**

... AND PROSPECTS

➤ Cosmic spectroscopy computed for:

- ❖ Single-exchange diagram. What about
What about exchange of spinning fields?



- ❖ Quadratic coupling ω treated perturbatively. What about large mixing?

↳ [Werth, Pinol, Renaux-Petel *in prep.*]

- ❖ Observational constraints and forecast. What are the current constraints? How better will we do with LSS?

➤ PNGs and GWs:

- ❖ Realistic trispectrum shape for models with small-scale enhancement and GWs contribution.
- ❖ Anisotropies from other soft limits of higher-order correlation functions

BACK UP SLIDES

ANISOTROPIES

SEVERAL SOURCES OF ANISOTROPIES

- GWs signal from astrophysical sources expected to be anisotropic
[Cusin *et al.* 2017, 2018, 2019]
[Bertacca *et al.* 2019]
[Bellomo *et al.* 2021]
- Cosmological background propagates through structures → anisotropic
[Alba, Maldacena 2015]
[Contaldi *et al.* 2016]
[Bartolo *et al.* 2018, 2019] *These anisotropies inherit a non-Gaussian statistics from propagation*
[Domcke, Jinno, Rubira 2020]
- Primordial NGs also induce anisotropies:
[Jeong, Kamionkowski 2012] *Anisotropies of the LSS from the same effect*
[Brahma, Nelson, Chandra 2013]
[Dimastrogiovanni *et al.* 2014, 2015, 2021]

PNG-INDUCED ANISOTROPIES

[Dimastrogiovanni, Fasiello, LP 2022]

ArXiv:2203.17192

- Formal derivations with the in-in formalism:

❖ We look for interactions between small and large scales $\rightarrow f_{\text{NL},\gamma\gamma\gamma}^{\text{sq}}$ and $f_{\text{NL},\gamma\gamma\zeta}^{\text{sq}}$

❖ A long-wavelength mode J_L can be treated classically and has negligible derivatives:

$$\hat{J}_L = J_L(\tau)\hat{a}_{\vec{k}} + J_L^*(\tau)\hat{a}_{-\vec{k}}^\dagger \rightarrow J_L^{\text{cl}}(\tau) \underbrace{(\hat{a}_{\vec{k}} + \hat{a}_{-\vec{k}}^\dagger)}_{\mathbf{b}_{\vec{k}}} \quad ; \quad (\partial_i J_L^{\text{cl}}, \partial_t J_L^{\text{cl}}) \text{ are negligible}$$

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}') \quad \mathbf{b}_{\vec{k}} \quad [b_{\vec{k}}, b_{\vec{k}'}^\dagger] = 0$$

Ex: massless scalar perturbation $Q_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k^3}} (1 + ik\tau) \xrightarrow{-k\tau \rightarrow 0} \frac{1}{\sqrt{2k^3}}$ purely real

PNG-INDUCED ANISOTROPIES

[Dimastrogiovanni, Fasiello, LP 2022]

ArXiv:2203.17192

- Formal derivations with the in-in formalism:

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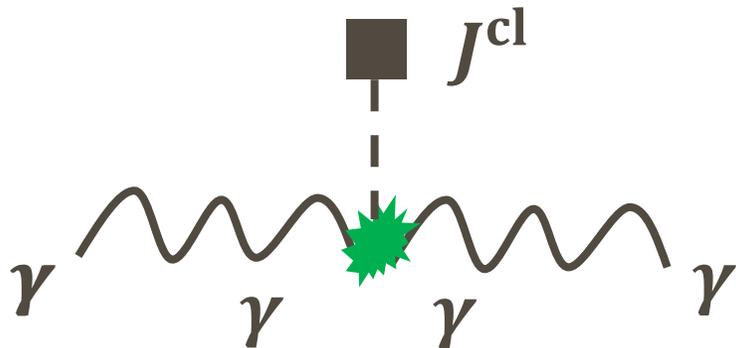
- ❖ A long-wavelength mode J_L can be treated classically and has negligible derivatives:

$$\hat{J}_L = J_L(\tau)\hat{a}_{\vec{k}} + J_L^*(\tau)\hat{a}_{-\vec{k}}^\dagger \rightarrow J_L^{\text{cl}}(\tau) \left(\hat{a}_{\vec{k}} + \hat{a}_{-\vec{k}}^\dagger \right) \quad ; \quad (\partial_i J_L^{\text{cl}}, \partial_t J_L^{\text{cl}}) \text{ are negligible}$$

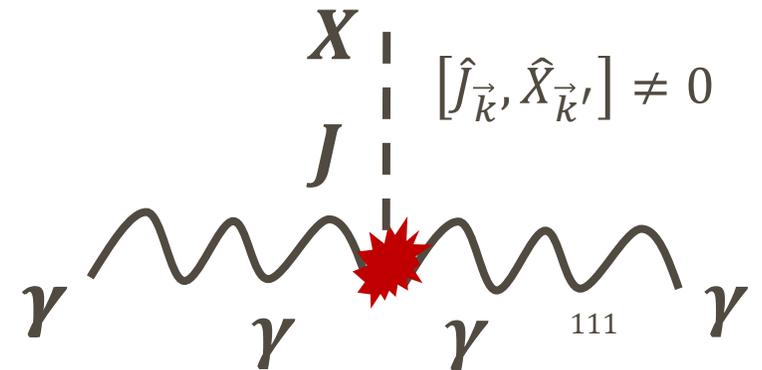
- ❖ A **3-pt interaction** involving J_L becomes a **2-pt interaction** times a classical source J_L^{cl}

- ❖ 2-pt functions in the presence of a classical source are now defined:

$$\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\text{cl}}}$$



$$\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} X_{\vec{k}_3} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B^{\gamma\gamma X}$$



PNG-INDUCED ANISOTROPIES

[Dimastrogiovanni, Fasiello, LP 2022]

ArXiv:2203.17192

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- ❖ A **3-pt interaction** involving J_L becomes a **2-pt interaction** times a classical source J_L^{cl}

- ❖ 2-pt functions in the presence of a classical source are now defined

- ❖ We compute both diagrams with the in-in formalism and are therefore able to relate them:

$$\langle \mathcal{Y}_{\vec{k}_1} \mathcal{Y}_{\vec{k}_2} \rangle_{X^{\text{cl}}} \Big|_{|\vec{k}_1 + \vec{k}_2| \ll k_{1,2}} = \int d^3 \vec{q} \delta^{(3)}(\vec{q} + \vec{k}_1 + \vec{k}_2) P_\gamma(k_1) f_{\text{NL},\text{sq}}^{\gamma\gamma X}(\vec{k}_1, \vec{k}_2, \vec{q}) X^{\text{cl}}(\vec{q})$$

Derivation makes clear that the non-diagonal part of the 2-pt function does not vanish \rightarrow anisotropies

J can be X (the formula reduces then to the one in the literature), or not but you need $[\hat{J}, \hat{X}] \neq 0^{112}$

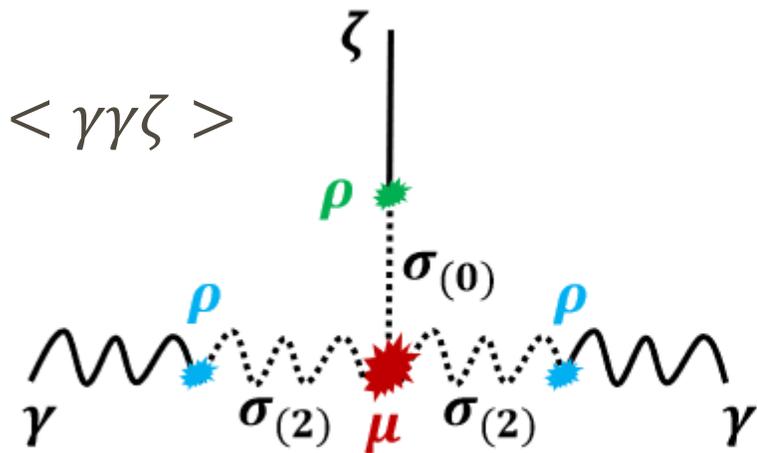
MULTIFIELD MODELS WITH LARGE ANISOTROPIES

- Spin-2 EFT of inflation: $\sigma_{ij} = \partial_i \partial_j \sigma^{(0)} + \sigma_{ij}^{(2)}$ [Bordin *et al.* 2018]

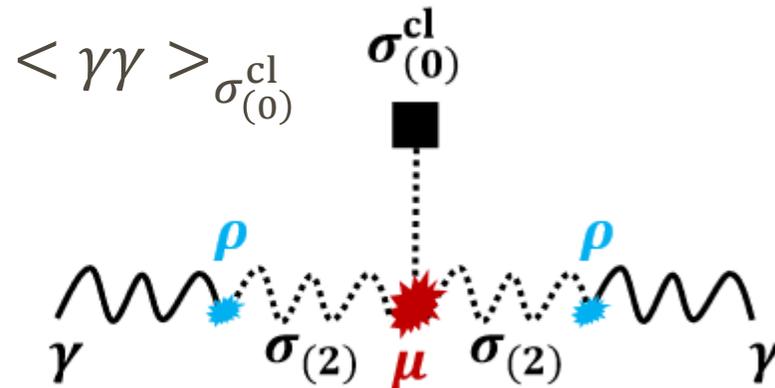
→ $\sigma^{(2)}$ couples linearly to γ and can enhance the tensor power spectrum: $A_t / \frac{2H^2}{M_{Pl}^2} \sim \frac{\rho^2}{c_2^3}$

make the tilt blue: $n_t \sim -3 \partial_t c_2 / (H c_2)$

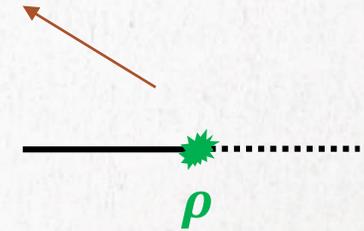
We compute anisotropies explicitly and find: $\sqrt{\langle \delta_{GW}^2(k_S, \hat{n}) \rangle} \sim \frac{\langle \gamma \gamma \zeta \rangle(k_S, k_S, k_L)}{P_\gamma(k_S) P_{\zeta \sigma^{(0)}}(k_L)} \sqrt{\mathcal{P}_{\sigma^{(0)}}(k_L)}$



(a) Mixed scalar-tensor-tensor bispectrum.



(b) Tensor two-point function in the presence of a classical scalar source.



[Dimastrogiovanni,
Fasiello, LP 2022]

ArXiv:2203.17192

MULTIFIELD MODELS WITH LARGE ANISOTROPIES

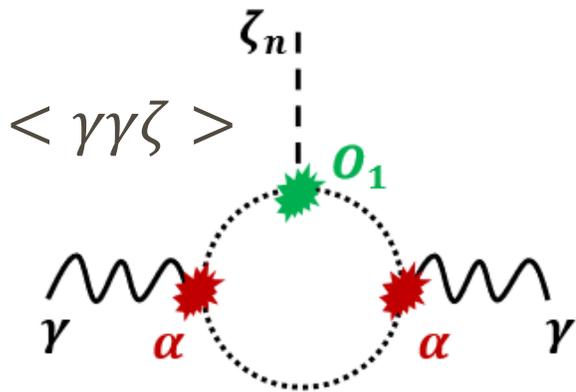
adiabatic entropic

- Supersolid inflation: two fundamental scalar fluctuations (ζ_n, R_{π_0}) [Celoria et al. 2021]

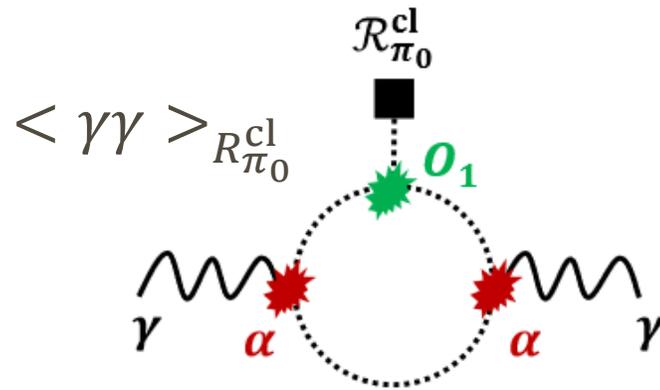
→ R_{π_0} couples **quadratically** to γ and can enhance the tensor power spectrum: $A_t / \frac{2H^2}{M_{Pl}^2} > 1$

make the tilt blue: $n_t = 2(n_s^{en} - 1) > 0$

We compute anisotropies explicitly and find: $\sqrt{\langle \delta_{GW}^2(k_S, \hat{n}) \rangle} \sim \underbrace{f_{NL,sq}^{\gamma\gamma\zeta_n}(k_S, k_S, k_L)}_{\gg 1} \left(\underbrace{\sqrt{\frac{\mathcal{P}_{\zeta_n} \mathcal{P}_{R_{\pi_0}}}{\mathcal{P}_{\zeta_n R_{\pi_0}}}}}_{O(1)} \right)^{k_L} \underbrace{A_s^{1/2}}_{4 \times 10^{-5}}$



(b) One-loop scalar-tensor-tensor bispectrum



(c) One-loop tensor two-point function in the presence of a classical scalar source.

[Dimastrogiovanni, Fasiello, LP 2022]

ArXiv:2203.17192

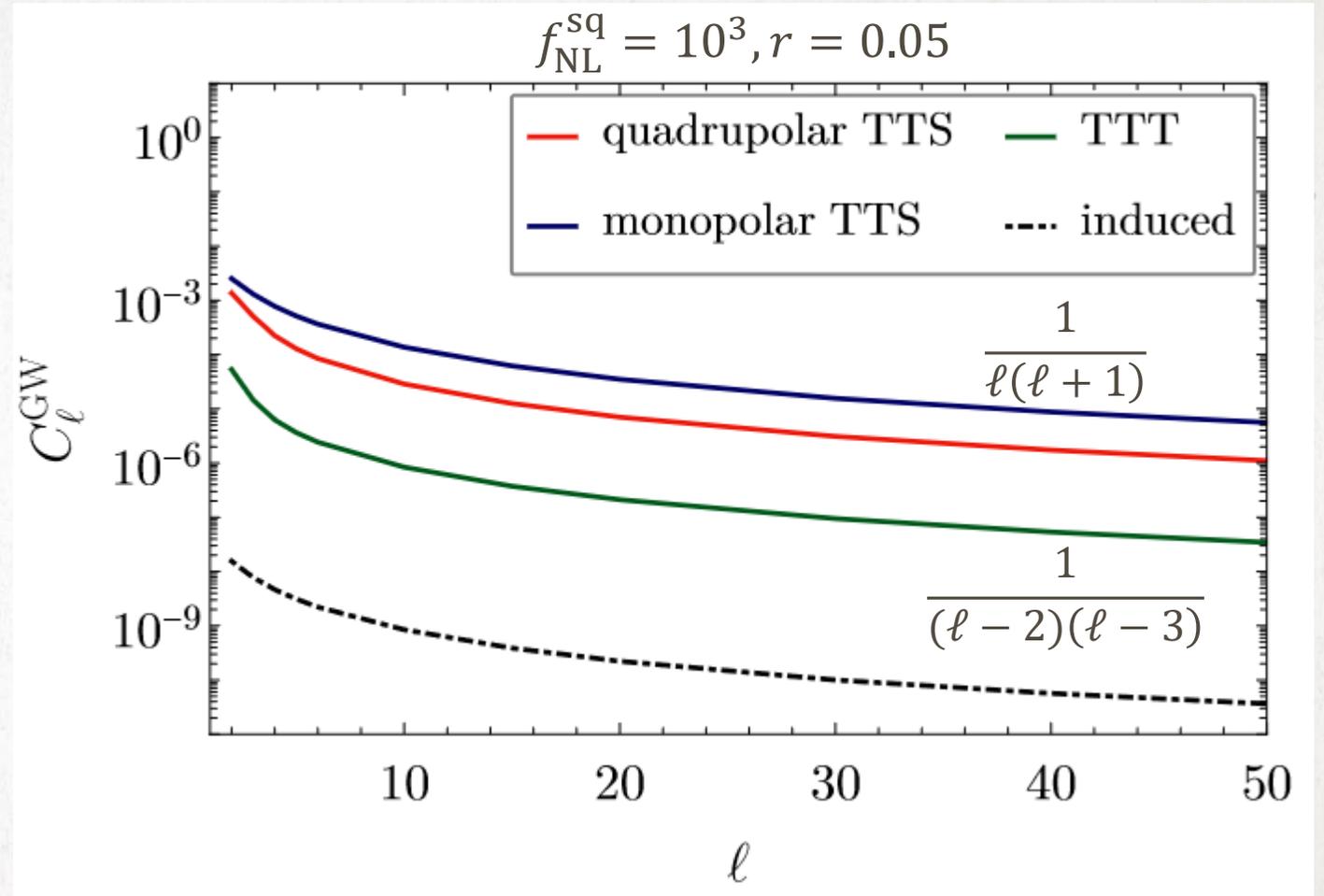
ℓ -DEPENDENCE

Anisotropies:

$$\Omega_{GW}(f, \hat{n}) = \bar{\Omega}_{GW}(f)(1 + \delta_{GW}(f, \hat{n}))$$

$$a_{\ell,m} = \int d\Omega Y_{\ell,m}(\hat{n}) \delta_{GW}(\hat{n})$$

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_m a_{\ell,m}^* a_{\ell,m}$$

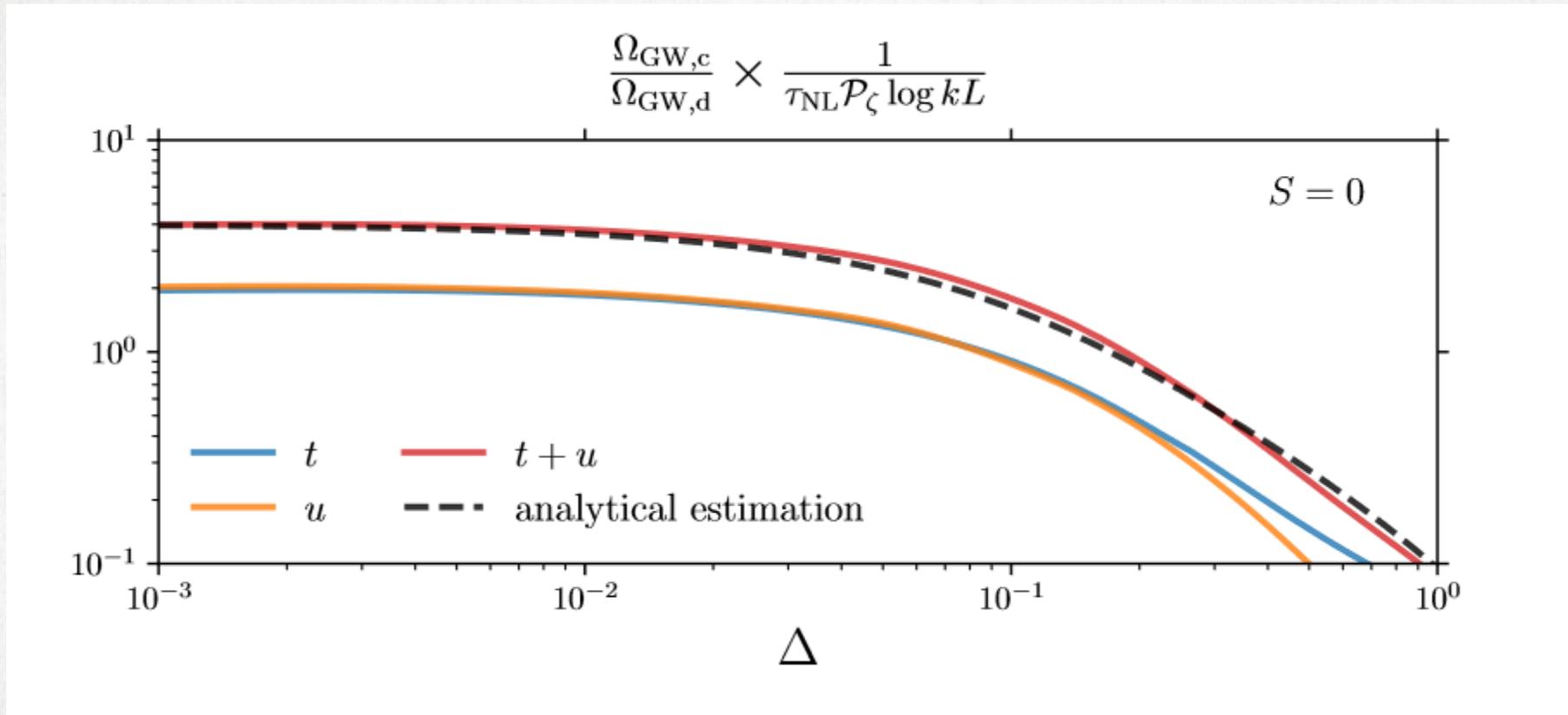


[Dimastrogiovanni *et al.* 2021]

BACK UP SLIDES

**TRISPECTRUM
INDUCED**

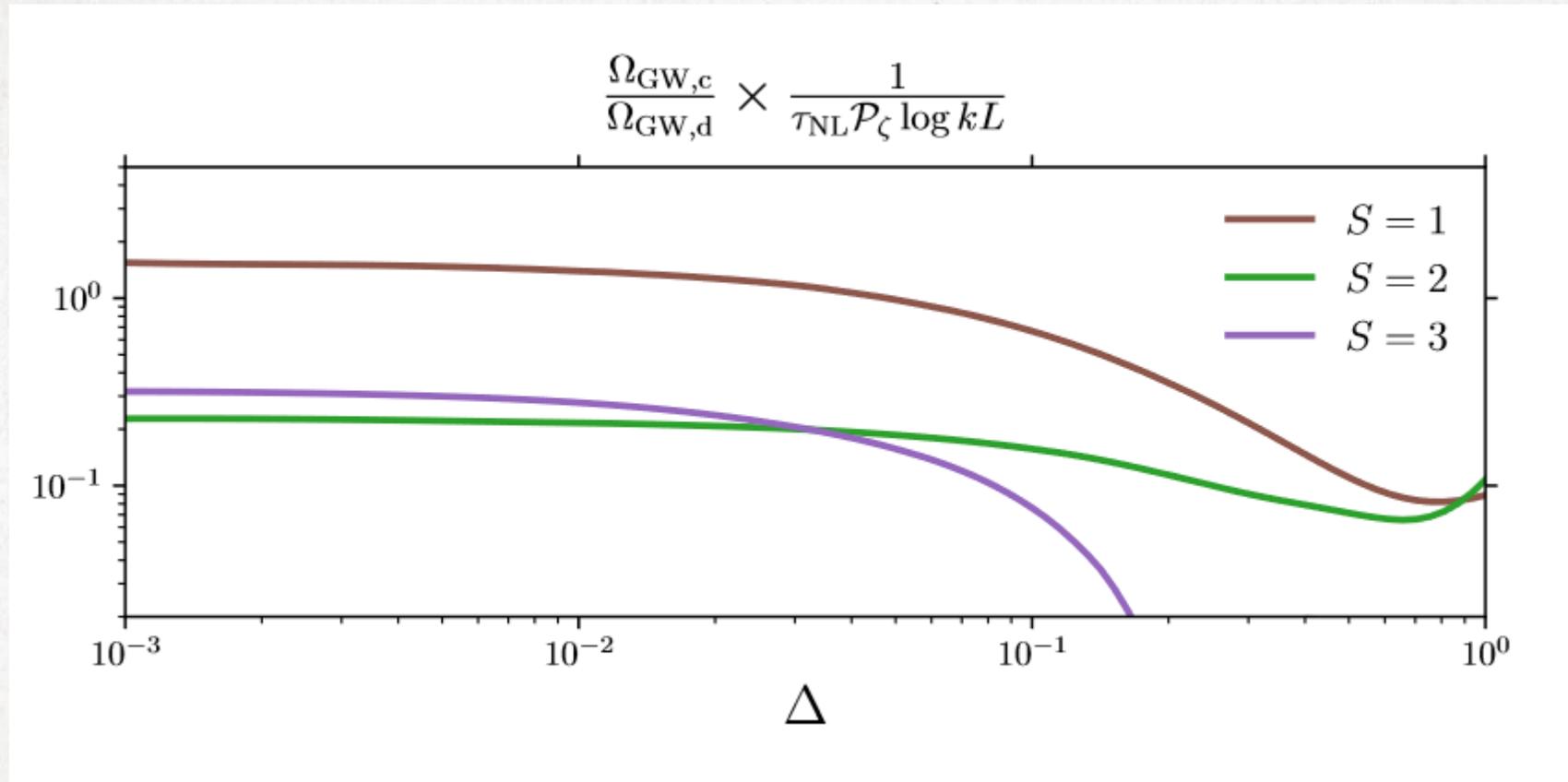
EXCHANGE OF A MASSIVE SCALAR FIELD



Analytical estimation:

$$\frac{\Omega_{\text{GW},c}}{\Omega_{\text{GW},d}} \simeq 4 \tau_{\text{NL}} \mathcal{P}_\zeta \frac{\alpha^2 \Delta}{2\Delta} \left[1 - e^{-2\Delta \log kL} \right]$$

EXCHANGE OF A MASSIVE SPINNING FIELD



BACK UP SLIDES

SCALAR PNG

BISPECTRUM IN MULTIFIELD INFLATION

The squeezed limit as a cosmological collider

Remember the single-field result:

$$f_{\text{NL}}^{\text{squeezed}} \propto n_s - 1 \ll 1$$

universal relation



Two-field result:

[Chen, Wang 2009]

[Noumi, Yamaguchi, Yokoyama 2013] (one extra heavy field $m_s > 3H/2$, perturbatively coupled)

[Arkani-Hamed, Maldacena 2015]

[Arkani-Hamed, Baumann, Lee, Pimentel 2018]

BISPECTRUM IN MULTIFIELD INFLATION

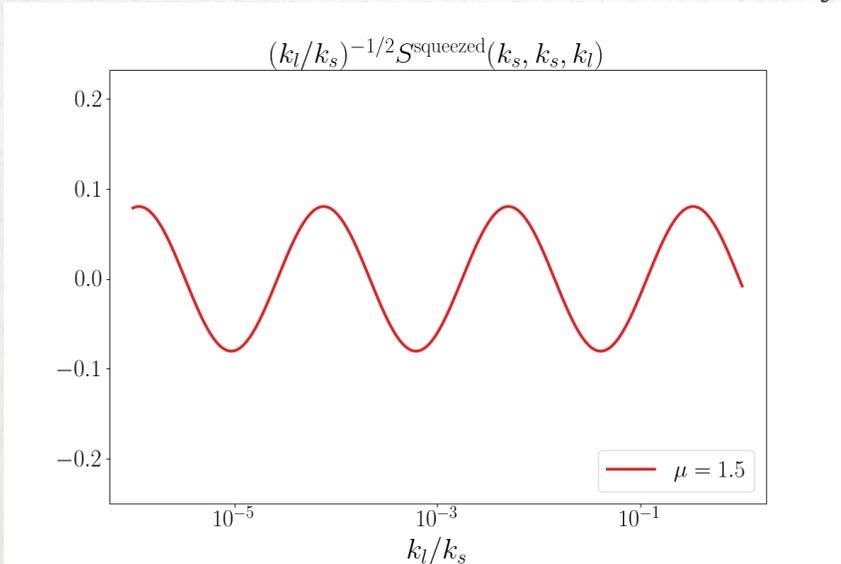
The squeezed limit as a cosmological collider

[Noumi, Yamaguchi,
Yokoyama 2013]

Two-field result: $f_{\text{NL}}^{\text{squeezed}} \simeq \eta_{\perp}^2 e^{-\pi\mu} \cos \left[\mu \log \left(\frac{k_l}{k_s} \right) \right]$ $k_l \ll k_s$

Small coupling between the two fields : $\eta_{\perp} \ll 1$





Oscillatory pattern: massive particle

$$\mu = \sqrt{\frac{m_s^2}{H^2} - \frac{9}{4}}$$

the reduced mass

Boltzmann suppression for heavy particles

$$H/2\pi \sim T \text{ so } e^{-\pi\mu} \sim e^{-\frac{m_s}{T}}$$

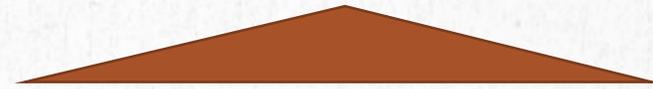
BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large coupling, $\eta_{\perp} \gg 1 \rightarrow$ Multifield instability \rightarrow Large flattened NGs:

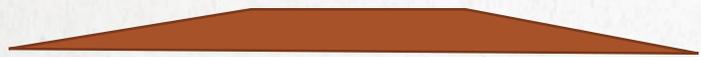
[Fumagalli, Garcia-Saenz, Lucas Pinol,
Renaux-Petel, Ronayne 2019]

Phys. Rev. Lett. 123, 201302



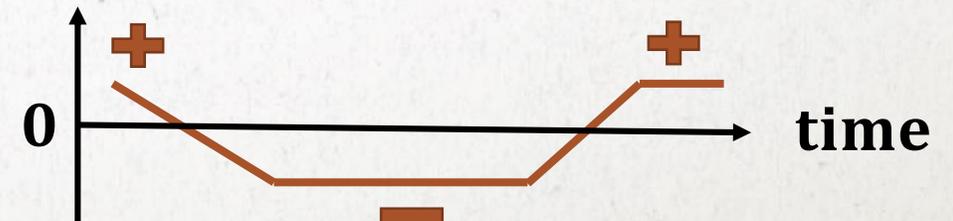
$$f_{\text{NL}}^{\text{flat}} = \mathcal{O}(50)$$

Higher-order correlation functions are boosted in similar configurations



$$g_{\text{NL}}^{\text{flat}} = \mathcal{O}(10^5) \text{ etc.}$$

$$m_{\text{eff}}^2/H^2$$



Clear sign of transiently unstable degrees of freedom:

BISPECTRUM IN MULTIFIELD INFLATION

Probing other regimes

- Large mass, $|m_s^2| \gg H^2 \rightarrow$ Single-field effective theory for ζ
(including the instability with $m_s^2 < 0$)

$$\text{L-shaped arrow} \rightarrow f_{\text{nl}}^{\text{eq}} \simeq \left(\frac{1}{c_s^2} - 1 \right) \left(-\frac{85}{324} + \frac{15}{243} A \right)$$

Speed of sound:

Dictated by the bilinear coupling η_{\perp}

[Achucarro, Gong, Hardeman, Palma, Patil 2012]

Single-field effective interactions

Dictated by the multifield cubic interactions

[Garcia-Saenz, Lucas Pinol, Renaux-Petel 2019]

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